Online Appendix to Incentive Contracts and the Allocation of Talent (Not for Publication)

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December 2015

1 Appendix B. Additional Figures

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C1. Proof of Lemma 5.

Proof. Combining the minimum firm size $\underline{n} = \frac{\sigma-1}{w}\frac{-1+\sqrt{1+2w\xi\Phi^2}}{\xi\Phi^2}$ (Equation 13 in the main text) and the wage function $w = \frac{\sigma-1}{\sigma}t\underline{a}\underline{n}^{-\frac{1}{\sigma}}$ (Equation 17 in the main text) yields the following free-entry condition

$$\Lambda \equiv (\frac{-1+\sqrt{1+2w\xi\Phi^2}}{\xi\Phi^2})^{\frac{1}{\sigma}} w^{1-\frac{1}{\sigma}} = \frac{(\sigma-1)^{1-\frac{1}{\sigma}}\underline{a}}{\sigma}t. \tag{C1}$$

Rewrite the market-clear condition (12) as

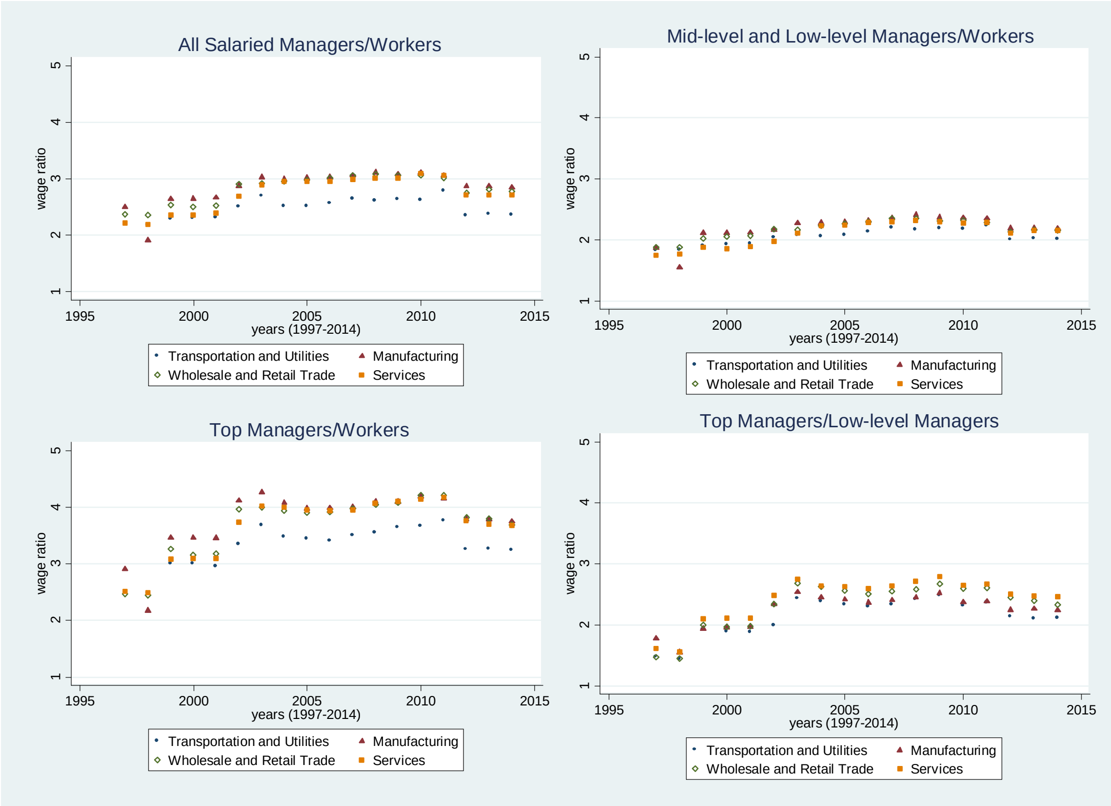
$$\Theta \equiv -\int_0^{\underline{a}} g(a)da + \underline{n}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma}g(a)da + \Phi\{\frac{\xi\Phi}{\sigma-1}w\underline{n}^2\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma}g(a)da \tag{C2}$$

$$+\sqrt{2\xi(w-\underline{w})}\underline{n}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma}g(a)da + \frac{\xi\Phi}{2(\sigma-1)}w\underline{n}^2\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma}g(a)da\} = 0.$$

These two equations implicitly define two unknown variables $(\underline{a}, w)$ in terms of the parameters including t. Totally differentiate (C1) and (C2) with respect to t:

$$\begin{bmatrix} \frac{d\Lambda}{dw} & -\frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t \\ \\ \frac{\partial\Theta}{\partial w} & \frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a}) \end{bmatrix} \begin{bmatrix} \frac{dw}{dt} \\ \\ \frac{d\underline{a}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}\underline{a} \\ \\ 0 \end{bmatrix}.$$

Figure B1. Wage Income by Manager Type and Industries



Notes: The wage ratio is defined as the ratio of the annual average wage of top managers over the annual average wage of workers. Workers include both production and clerical workers. Low-level managers are individuals whose occupations are described as "first-line supervisors" or "administrative managers." Top managers are individuals whose occupations are described as "general managers" or "chief executives." Mid-level managers are individuals whose occupations belong to the managerial category, but are not described as those of the low-level managers or top managers. All salaried managers are the sum of the above three types of managers. Industries are grouped based on SIC or NAICS codes. Data source: US. National Occupational Compensation Statistics, U.S. Bureau of Labor Statistics

Figure B2. Employment by Manager Type and Industries

Notes: Self-employed workers are individuals who are classified as "Self-employed" in the U.S. Labor Statistics. Salaried managers are individuals whose occupation is classified as "Managerial occupation" in the U.S. National Occupational Compensation Statistics. Top managers are individuals whose occupations are described as "general managers" or "chief executives." All managers include both the self-employed and salaried managers. Data source: U.S. Bureau of Labor Statistics.

Then,

$$\begin{bmatrix} \frac{dw}{dt} \\ \\ \frac{d\underline{a}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}\underline{a}}{[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{d\Lambda}{dw} + \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w}} \\ \\ \frac{-\frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}\underline{a}\frac{\partial\Theta}{\partial w}}{[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{d\Lambda}{dw} + \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w}} \end{bmatrix}. \tag{C3}$$

Using the proof of Lemma 4, we can show that $\frac{\partial\Theta}{\partial\underline{a}} < 0$ and thus $\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a}) < 0$. We now consider the sign of $\frac{\partial\Theta}{\partial w}$ and that of $[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{d\Lambda}{dw} + \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w}$.

Since the terms involving the derivatives of the threshold values of talent cancel out,

$$\begin{aligned}
\frac{\partial\Theta}{\partial w} = {} & \frac{d\underline{n}}{dw}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma}g(a)da + (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)[\\
& \frac{\xi\Phi^2\underline{n}^2}{\sigma-1}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma}g(a)da + \frac{\xi\Phi\underline{n}}{\sqrt{2\xi w}}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma}g(a)da + \frac{\xi\Phi^2\underline{n}^2}{2(\sigma-1)}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma}g(a)da] \\
= {} & \int_{\underline{a}}^{a^*}\underbrace{[\frac{d\underline{n}}{dw}(\frac{a}{\underline{a}})^{\sigma} + (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{\sigma-1}(\frac{a}{\underline{a}})^{2\sigma}]}_{\Theta_1}g(a)da \\
& + \underbrace{[\frac{d\underline{n}}{dw} + (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi\underline{n}}{\sqrt{2\xi w}}]}_{\Theta_2}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma}g(a)da \\
& + \int_{a^{**}}^{\infty}\underbrace{[\frac{d\underline{n}}{dw}(\frac{a}{\underline{a}})^{\sigma} + (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{2(\sigma-1)}(\frac{a}{\underline{a}})^{2\sigma}]}_{\Theta_3}g(a)da.
\end{aligned}$$

In what follows, we show $\Theta_1 > 0, \Theta_2 > 0$, and $\Theta_3 > 0$.

Calculate

$$\frac{d\underline{n}}{dw} = \frac{\sigma-1}{\xi w^2\Phi^2}\frac{\sqrt{1+2w\xi\Phi^2}-(1+w\xi\Phi^2)}{\sqrt{1+2w\xi\Phi^2}} < 0;$$

and

$$\begin{aligned}
& 2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1 \\
& = \frac{-2\frac{1+w\xi\Phi^2}{\sqrt{1+2w\xi\Phi^2}} + 2 + \sqrt{1+2w\xi\Phi^2} - 1}{\sqrt{1+2w\xi\Phi^2}-1} \\
& = \frac{1}{\sqrt{2\xi w\Phi^2+1}} > 0.
\end{aligned}$$

Since

$$\begin{aligned}
&\frac{d\underline{n}}{dw}+(2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{\sigma-1}\\
&=\frac{\sigma-1}{\xi w^2\Phi^2}\frac{\sqrt{1+2w\xi\Phi^2}-(1+w\xi\Phi^2)}{\sqrt{1+2w\xi\Phi^2}}+\frac{1}{\sqrt{2\xi w\Phi^2+1}}\frac{\xi\Phi^2\underline{n}^2}{\sigma-1}\\
&=\frac{(\sigma-1)(1+\xi w\Phi^2-\sqrt{2\xi w\Phi^2+1})}{\xi w^2\Phi^2\sqrt{1+2w\xi\Phi^2}}>0,
\end{aligned}$$

$$\Theta_1>0 \text{ and } \Theta_3>0.$$

Consider

$$\begin{aligned}
\Theta_2&\equiv\frac{d\underline{n}}{dw}+(2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi\underline{n}}{\sqrt{2\xi w}}\\
&=\frac{\sigma-1}{\xi w^2\Phi^2}\frac{\sqrt{1+2w\xi\Phi^2}-(1+w\xi\Phi^2)}{\sqrt{1+2w\xi\Phi^2}}+\frac{\sigma-1}{w\sqrt{2\xi w\Phi^2}}\frac{\sqrt{1+2w\xi\Phi^2}-1}{\sqrt{1+2w\xi\Phi^2}}\\
&=\frac{\sigma-1}{2\xi w^2\Phi^2}\frac{2\sqrt{1+2w\xi\Phi^2}-2(1+w\xi\Phi^2)+\sqrt{2\xi w\Phi^2}(\sqrt{1+2w\xi\Phi^2}-1)}{\sqrt{1+2w\xi\Phi^2}}.
\end{aligned}$$

Let $2w\xi\Phi^2=x$ and define a function $h(x)=2\sqrt{1+x}-2-x+\sqrt{1+x}\sqrt{x}-\sqrt{x}$.

$$\begin{aligned}
\frac{dh(x)}{dx}&=\frac{2}{2\sqrt{1+x}}-1+\frac{\sqrt{1+x}}{2\sqrt{x}}+\frac{\sqrt{x}}{2\sqrt{1+x}}-\frac{1}{2\sqrt{x}}\\
&=\frac{2x+2\sqrt{x}+1-(2\sqrt{x}+1)\sqrt{x+1}}{2\sqrt{x}\sqrt{x+1}}>0.
\end{aligned}$$

Given that $h(0)=0$, $h(x)>0$ for $x>0$. Then

$$\Theta_2>0.$$

Therefore,

$$\frac{\partial\Theta}{\partial w}>0.$$

Now, we are left with the task of determining the sign of $[\frac{\partial\Theta}{\partial\underline{a}}-g(\underline{a})]\frac{d\Lambda}{dw}+\frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w}$.

Since $\frac{d\Lambda}{dw} = \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}\underline{a}t[\frac{(\sigma-1)}{\sigma w\underline{n}\sqrt{1+2\xi w\Phi^2}} + \frac{\sigma-1}{\sigma w}]$,

$$
\begin{aligned}
&[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{d\Lambda}{dw} + \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w} \\
&= \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\{[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{\underline{a}(\sigma-1)}{\sigma w} + \frac{-g(\underline{a})\underline{a}(\sigma-1)}{\sigma w\underline{n}\sqrt{1+2\xi w\Phi^2}} \\
&+ [\frac{\underline{a}(\sigma-1)}{\sigma w\underline{n}\sqrt{1+2\xi w\Phi^2}}\frac{\partial\Theta}{\partial\underline{a}} + \frac{\partial\Theta}{\partial w}]\}.
\end{aligned}
$$

Obviously, the first two terms in the curly bracket are negative; if the third term is negative as well, the whole expression will be negative. Following the proof of Lemma 4, we write

$$
\begin{aligned}
\frac{\partial\Theta}{\partial\underline{a}} &= -\underline{n}g(\underline{a}) - \int_{\underline{a}}^{\infty}\frac{\sigma}{\underline{a}}(\frac{a}{\underline{a}})^{\sigma}\underline{n}g(a)da - \frac{\xi\Phi^2}{\sigma-1}w\underline{n}^2\int_{\underline{a}}^{a^*}\frac{2\sigma}{\underline{a}}(\frac{a}{\underline{a}})^{2\sigma}g(a)da \\
&- \sqrt{2\xi(w-\underline{w})}\underline{n}\frac{\sigma\Phi}{\underline{a}}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma}g(a)da - \frac{\xi\Phi^2}{2(\sigma-1)}w\underline{n}^2\int_{a^{**}}^{\infty}\frac{2\sigma}{\underline{a}}(\frac{a}{\underline{a}})^{2\sigma}g(a)da.
\end{aligned}
$$

Then,

$$
\begin{aligned}
&\frac{(\sigma-1)\underline{a}}{\sigma w\underline{n}\sqrt{1+2\xi w\Phi^2}}\frac{\partial\Theta}{\partial\underline{a}} + \frac{\partial\Theta}{\partial w} \\
&= -\underline{a}\frac{(\sigma-1)}{\sigma w\sqrt{1+2\xi w\Phi^2}}g(\underline{a}) - [\frac{(\sigma-1)}{w\sqrt{1+2\xi w\Phi^2}} - \frac{d\underline{n}}{dw}]\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma}g(a)da \\
&- \underbrace{[\frac{2\xi\Phi^2\underline{n}}{\sqrt{1+2\xi w\Phi^2}} - (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{\sigma-1}]}_{\Omega_1}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma}g(a)da \\
&- \underbrace{[\frac{(\sigma-1)\sqrt{2\xi w\Phi^2}}{w\sqrt{1+2\xi w\Phi^2}} - (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\Phi k\underline{n}}{\sqrt{2\xi w}}]}_{\Omega_2}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma}g(a)da \\
&- \underbrace{[\frac{\xi\Phi^2\underline{n}}{\sqrt{1+2\xi w\Phi^2}} - (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{2(\sigma-1)}]}_{\Omega_3}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma}g(a)da.
\end{aligned}
$$

Given that

$$\frac{2\xi\Phi^2\underline{n}}{\sqrt{1+2\xi w\Phi^2}} - (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\Phi^2\underline{n}^2}{\sigma-1}$$
$$= \frac{\xi\Phi^2\underline{n}(1+2\xi w\Phi^2-\sqrt{2\xi w\Phi^2+1})}{\xi w\Phi^2\sqrt{1+2\xi w\Phi^2}} > 0,$$

and

$$\frac{(\sigma-1)\sqrt{2\xi w\Phi^2}}{w\sqrt{1+2\xi w\Phi^2}} - (2\frac{w}{\underline{n}}\frac{d\underline{n}}{dw}+1)\frac{\xi\underline{n}}{\sqrt{2\xi w}}$$
$$= \frac{(\sigma-1)(1+2\xi w\Phi^2-\sqrt{2\xi w\Phi^2+1})}{w\Phi\sqrt{2\xi w}\sqrt{1+2\xi w\Phi^2}} > 0,$$

we have

$$\Omega_1 > 0, \Omega_2 > 0, \text{and } \Omega_3 > 0.$$

As a result,

$$\frac{(\sigma-1)\underline{a}}{\sigma w\underline{n}\sqrt{1+2\xi w\Phi^2}}\frac{\partial\Theta}{\partial\underline{a}} + \frac{\partial\Theta}{\partial w} < 0$$

and

$$[\frac{\partial\Theta}{\partial\underline{a}} - g(\underline{a})]\frac{d\Lambda_1}{dw} + \frac{(\sigma-1)^{1-\frac{1}{\sigma}}}{\sigma}t\frac{\partial\Theta}{\partial w} < 0.$$

Then, from (B3),

$$\frac{d\underline{a}}{dt} > 0 \text{ and } \frac{dw}{dt} > 0.$$

■

C2. Proof of Proposition 5.

Proof. Proposition 5 relies on the existence of a unique equilibrium in the monopolistic competition model, which we will prove now. The market clearing condition is

$$M\cdot f + \int_{\underline{a}}^{\infty}\frac{\widetilde{q(a)}}{a}g(a)da = 1-M, \tag{C4}$$

where $M \equiv \int_{\underline{a}}^{\infty} g(a)da$ is the number of managers and $M\cdot f$ is the aggregate demand for raw labor to bear the fixed costs; $\frac{\widetilde{q(a)}}{a} = e(a)\frac{q(\varphi a)}{\varphi a} + [1-e(a)]\frac{q(a)}{a}$ is the aggregate demand for raw labor to bear the variable costs. Rewrite (C4) as

$$\int_{\underline{a}}^{\infty} r(a)g(a)da + (\varphi^{\varepsilon-1}-1)\int_{\underline{a}}^{\infty} e(a)r(a)g(a)da = \frac{\varepsilon}{\varepsilon-1}[1-M(1+f)]. \tag{C5}$$

Using the relationship between revenues across firms and the corresponding managerial efforts, the left-hand side of (C5) becomes

$$\underline{r}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\varepsilon-1}g(a)da+(\varphi^{\varepsilon-1}-1)\{\underline{e}\cdot\underline{r}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da \tag{C6}$$

$$+\sqrt{2\xi(1-\underline{w})}\underline{r}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\varepsilon-1}g(a)da+\frac{\underline{e}\cdot\underline{r}}{2}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da\},$$

where $\underline{r}=\varepsilon\frac{-1+\sqrt{1+2\xi(\varphi^{\varepsilon-1}-1)^2(f+1)}}{\xi(\varphi^{\varepsilon-1}-1)^2}$ and $\underline{e}=\frac{-1+\sqrt{1+2\xi(\varphi^{\varepsilon-1}-1)^2(f+1)}}{2(\varphi^{\varepsilon-1}-1)}$.

The first term of (C6) is decreasing in $\underline{a}$ since $\frac{d\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\varepsilon-1}g(a)da}{d\underline{a}}=-g(\underline{a})-\frac{\varepsilon-1}{\underline{a}}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\varepsilon}g(a)da<0$.

Denote the terms in curly bracket $\Phi(\underline{a})$. Then

$$\Phi'(\underline{a})=\underline{e}\cdot\underline{r}[\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{2\sigma-2}g(a^*)-g(\underline{a})-\frac{2\sigma-2}{\underline{a}}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da] \tag{C7}$$

$$+\sqrt{2\xi(1-\underline{w})}\underline{r}[\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1}g(a^{**})-\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\varepsilon-1}g(a^*)-\frac{\varepsilon-1}{\underline{a}}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\varepsilon-1}g(a)da]$$

$$+\frac{\underline{e}\cdot\underline{r}}{2}[-\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{2\varepsilon-2}g(a^{**})-\frac{2\varepsilon-2}{\underline{a}}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\varepsilon-2}g(a)da].$$

Since $\frac{a^*}{\underline{a}}=[\frac{\sqrt{2\xi(1-\underline{w})}}{\underline{e}}]^{\frac{1}{\varepsilon-1}}$, the terms

$$\underline{e}\cdot\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{2\sigma-2}g(a^*)-\sqrt{2\xi(1-\underline{w})}\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\varepsilon-1}g(a^*)$$
$$=\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\varepsilon-1}g(a^*)[\underline{e}(\frac{a^*}{\underline{a}})^{\varepsilon-1}-\sqrt{2\xi(1-\underline{w})}]=0.$$

Since $\frac{a^{**}}{\underline{a}}=[\frac{2\sqrt{2\xi(1-\underline{w})}}{\underline{e}}]^{\frac{1}{\varepsilon-1}}$, the terms

$$\sqrt{2\xi(1-\underline{w})}\underline{r}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1}g(a^{**})-\frac{\underline{e}\cdot\underline{r}}{2}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{2\sigma-2}g(a^{**})$$
$$=\underline{r}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1}g(a^{**})[\sqrt{2\xi(1-\underline{w})}-\frac{\underline{e}}{2}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1}]=0.$$

All the other terms in (C7) are negative. Hence, $\Phi'(\underline{a})<0$: the demand curve is downward sloping. Since $M = \int_{\underline{a}}^{\infty} g(a)da$ decreases in $\underline{a}$, the right-hand side of (C5) increases in $\underline{a}$. Moreover, the difference between the left-hand side and the right-hand side is positive when $\underline{a} \to 0$, and the difference is negative when $\underline{a} \to \infty$. By the intermediate value theorem, there exists a single interior point $\underline{a} \in (0, \infty)$ that equates the two sides of (C5). Therefore, a unique cutoff talent $\underline{a}$ is well defined.

We now turn to the proof of Proposition 5.

Part 1). The first result holds for any continuous and differential probability distributions. Thus, I prove a general case without specifying a Pareto distribution of talent. Rewrite (C4) as an implicit function in $(\underline{a}, \varepsilon)$:

$$\begin{aligned}\Omega(\underline{a}, \varepsilon) = & \frac{e}{\xi(\varphi^{\varepsilon-1} - 1)} \int_{\underline{a}}^{\infty} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \frac{e^2}{\xi} \int_{\underline{a}}^{a^*} (\frac{a}{\underline{a}})^{2(\varepsilon-1)} g(a)da \\ & + \sqrt{2\xi(1-\underline{w})} \frac{e}{\xi} \int_{a^*}^{a^{**}} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \frac{e^2}{2\xi} \int_{a^{**}}^{\infty} (\frac{a}{\underline{a}})^{2(\varepsilon-1)} g(a)da \\ & - \frac{\varepsilon}{\varepsilon - 1} [1 - (1+f) \int_{\underline{a}}^{\infty} g(a)da] = 0.\end{aligned}$$

Add a term $\frac{e^2}{2\xi} \int_{\underline{a}}^{\infty} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da$ and subtract the same value $\frac{e^2}{2\xi}[\int_{\underline{a}}^{a^*} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \int_{a^*}^{a^{**}} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \int_{a^{**}}^{\infty} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da]$. Rearranging terms and using the market entry condition: $\frac{e^2}{2\xi} + \frac{e}{\xi(\varphi^{\varepsilon-1}-1)} = 1 + f$, $\Omega(\underline{a}, \varepsilon)$ can be written as:

$$\begin{aligned}& (1+f) \int_{\underline{a}}^{\infty} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \frac{e^2}{\xi} \int_{\underline{a}}^{a^*} [(\frac{a}{\underline{a}})^{2(\varepsilon-1)} - \frac{1}{2}(\frac{a}{\underline{a}})^{\varepsilon-1}] g(a)da \\ & + [\sqrt{2\xi(1-\underline{w})} \frac{e}{\xi} - \frac{e^2}{2\xi}] \int_{a^*}^{a^{**}} (\frac{a}{\underline{a}})^{\varepsilon-1} g(a)da + \frac{e^2}{2\xi} \int_{a^{**}}^{\infty} [(\frac{a}{\underline{a}})^{2(\varepsilon-1)} - (\frac{a}{\underline{a}})^{\varepsilon-1}] g(a)da \\ & - \frac{\varepsilon}{\varepsilon - 1} [1 - (1+f) \int_{\underline{a}}^{\infty} g(a)da].\end{aligned}$$

Holding $\underline{a}$ constant, differentiate $\Omega(\underline{a}, \varepsilon)$ with respect to ε. It is straightforward to show that $\Omega(\underline{a}, \varepsilon)$ increases in ε if a^* and a^{**} are held as constant. The key is to check the terms involving the derivatives of the boundary values of the integral (a^* and a^{**}), which are collected as:

$$\underbrace{[\frac{\underline{e}^2}{\xi}(\frac{a^*}{\underline{a}})^{\varepsilon-1} - \frac{\underline{e}^2}{2\xi} - \frac{2\sqrt{2\xi(1-\underline{w})}\underline{e} - \underline{e}^2}{2\xi}]}_{=0}(\frac{a^*}{\underline{a}})^{\varepsilon-1} g(a^*)\frac{da^*}{d\sigma}$$
$$+ \underbrace{[\frac{2\sqrt{2\xi(1-\underline{w})}\underline{e} - \underline{e}^2}{2\xi} + \frac{\underline{e}^2}{2\xi} - \frac{\underline{e}^2}{2\xi}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1}]}_{=0}(\frac{a^{**}}{\underline{a}})^{\varepsilon-1} g(a^{**})\frac{da^{**}}{d\sigma} = 0.$$

The effects through the threshold values cancel out. Hence $\frac{\partial\Omega(\underline{a},\varepsilon)}{\partial\varepsilon} > 0$. From the proof of existence of the equilibrium, we can easily obtain $\frac{\partial\Omega(\underline{a},\varepsilon)}{\partial\underline{a}} < 0$. Therefore

$$\frac{d\underline{a}}{d\sigma} = -\frac{\frac{\partial\Omega(\underline{a},\varepsilon)}{\partial\varepsilon}}{\frac{\partial\Omega(\underline{a},\varepsilon)}{\partial\underline{a}}} > 0.$$

Part 2) With the Pareto distribution, the fractions of each type of manager among all managers are

$$\underline{\theta} = 1 - (\frac{a^*}{\underline{a}})^{-\lambda};$$
$$\theta^* = (\frac{a^*}{\underline{a}})^{-\lambda}(1 - 2^{\frac{-\lambda}{\varepsilon-1}});$$
$$\theta^{**} = 2^{\frac{-\lambda}{\varepsilon-1}}(\frac{a^*}{\underline{a}})^{-\lambda}.$$

Since $\frac{d\underline{e}}{d\sigma} > 0$ and $\frac{d\frac{-\lambda}{\varepsilon-1}}{d\sigma} > 0$, $(\frac{a^*}{\underline{a}})^{-\lambda} = [\frac{\sqrt{2\xi(1-\underline{w})}}{\underline{e}}]^{\frac{-\lambda}{\varepsilon-1}}$ decreases in ε. Hence, $\frac{d\underline{\theta}}{d\sigma} < 0$ and $\frac{d\theta^{**}}{d\sigma} > 0$.

Part 3) With the Pareto distribution, the expected wage of a manager receiving a rent-sharing incentive contract consists of two components: 1) a constant $\underline{w}$ and 2) a variable component drawn from a Pareto distribution with a shape parameter $\frac{\lambda}{2(\varepsilon-1)}$ and with a minimum value $2(1-\underline{w})$. Since $\frac{\lambda}{2(\varepsilon-1)}$ decreases in ε, the wage distribution of the managers with high-powered pay features a larger variance, a higher skewness, and thus a greater wage inequality. The average compensation to these managers is

$$\widetilde{W}^{rent} = \underline{w} + 2(1-\underline{w})\frac{\lambda}{\lambda - 2(\varepsilon-1)},$$

which increases in ε.

■