

Incentive Contracts and the Allocation of Talent

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Abstract

This paper develops a theory of sorting that links ability, pay-performance sensitivity, and pay levels across a wide range of managerial levels. Firms employ managers to improve productivity. Because of limited liability, firms use incentive contracts to elicit unobservable managerial effort; the type of optimal contract depends on a manager's ability. In equilibrium, individuals are sorted based on ability into production workers, business owners, managers paid an ability-invariant bonus, and managers whose pay varies with ability and firm size. The model predicts that TFP-enhancing technological progress and increased competition in product markets tend to 1) reduce the fraction of business owners while increasing the fraction of profit-sharing managers and 2) increase wage inequality between workers and managers and wage inequality among managers. Wage dynamics and employment of the self-employed and various groups of salaried managers in the U.S. are consistent with these predictions.

Key Words: Allocation of Talent, Incentive Contracts, Limited liability, Pay Structure, Wage Distribution

JEL Classifications: D2, J3, L1, L2, M5

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1 Introduction

Economists have long been interested in the relationship between individual ability and pay-performance sensitivity. Lazear (1986)’s well-known theory posits that high-ability workers sort into jobs with performance pay while low-ability workers take jobs with fixed wages. A related strand of literature argues that CEOs earn high levels of compensation primarily because of their high-powered performance incentives.¹ Although these theories offer a plausible explanation for compensation patterns among executive managers of large corporations, their explanatory power regarding compensation practices outside the executive suite is less obvious. For instance, owners of small businesses such as farms, grocery stores, laundry shops, and local bakeries are residual claimants with high-powered compensation schemes, yet they earn less than an average mid-level manager.² From the perspective of received theory, it is also puzzling why production and clerical workers receive roughly the same overall compensation regardless of whether they have performance incentives, and why the pay of mid-level salaried managers does not appear to differ significantly across skill levels.³

This paper develops a theory that links worker ability, pay-performance sensitivity, and pay level across a wide range of jobs. The theory accounts not only for patterns in top managers’ compensation, but also for compensation practices involving mid-level managers, small business owners, and nonmanagement “production” workers. The model integrates a principal-agent problem into a Lucas (1978) general equilibrium framework. In equilibrium, individuals are sorted based on managerial ability into production workers, small business owners, salaried managers whose pay does not vary with managerial ability, and managers whose pay varies with ability and also with firm profits. This sorting pattern produces a relationship between incentive structure and pay level that is not only consistent with prior studies of top managers but also sheds light on the wage distribution among lower-level managers and other workers.

Another novelty of the paper is that it provides a framework to study the effects of technological progress and market competition on both the wages and employment of various groups of individuals. The model predicts that a TFP-enhancing technological improvement and stiffer competition in the product market tend to select more-talented individuals into the managerial occupation, decrease the share of small business owners, increase the share of managers who share firm profits, and ultimately result in a more-skewed wage distribution. I show that the time-series and cross-industry patterns of wages and employment in the U.S.

¹See the surveys of Murphy (1999), Aggarwal (2008), and Frydman and Jenter (2010) and references therein.

²Based on a large-scale survey in 2014, the average annual wage of a restaurant owner in the U.S. was approximately \$60,000, a retail or grocery store owner \$52,000, and a hair salon or SPA owner \$40,000. By contrast, the average annual wage of an operations manager was approximately \$60,300, a regional sales manager \$75,000, and a manufacturing plant manager \$82,000. Data source: www.PayScale.com.

³The U.S. Occupational Survey data show that among production workers, the ones paid by time earn \$15.77 per hour while the ones paid by incentives earn \$16.03; the wage difference between these two types of workers among clerical and administrative workers is even smaller. See Table 1 for more detail.

during the last two decades are consistent with these theoretical predictions.

The model is built on three key elements: 1) a managerial firm in which a manager's talent and effort complement one another in production, 2) contractual frictions due to limited liability, and 3) a competitive product market. The economy-wide equilibrium is an allocation in which individuals with heterogeneous talent are sorted into a variety of employment statuses, exerting effort based on the optimal incentive structures offered by their employers and in which firms determine their employment and production levels to clear the labour and product markets.

The main intuition for the results presented herein is that more-talented managers create greater value for firms, which then use incentive contracts that incur higher costs to motivate managers. Specifically, firms employ managers to improve productivity. The limited-liability constraint restricts a firm's ability to punish managerial slack, creating an agency problem with regard to unobservable managerial effort. A low-talent manager creates a small amount of surplus, and the limited-liability constraint is slack. Thus, the most efficient manner to align managerial incentives is to transfer ownership to the manager who can afford the surplus with future income. More-talented managers create larger surplus, and the limited-liability constraint becomes binding, preventing ownership transfer. Their employers then design a certain form of contingent pay to mitigate managerial slack. For a medium-talent manager, the surplus created is not sufficient to induce the firm to share rent with the manager; the optimal incentive structure is a bonus that is determined by the manager's outside option and, is thus independent of firm profitability. By contrast, the effort of a high-talent manager yields sufficiently large surplus that the owner optimally offers a rent-sharing contract.

In equilibrium, the population is stratified into four groups: 1) the least-talented individuals who are production workers receiving an economy-wide flat wage; 2) low-talent individuals who are residual claimants; 3) medium-talent individuals who are salaried managers receiving an ability-invariant bonus; and 4) high-talent individuals who are salaried managers sharing rent with their employers. This sorting pattern enriches the Lucas (1978)'s model, in which low-talent individuals are workers and high-talent individuals become managers who are also residual claimants.

The equilibrium sorting pattern in this model delivers three testable implications regarding the distribution of incentive structure and pay level. First, the relationship between managerial talent and the power of incentives is non-assortative. At the low end of talent distribution, the incentives are extremely low-powered (production workers), immediately followed by extremely high-powered incentives (residual claimants), and then by an incentive structure with intermediate power (salaried managers). Second, within the managerial class, the efficiency of incentive declines weakly with talent, whereas the size of incentive increases weakly with talent. This matches the empirical finding that managerial incentives increase substantially with firm size, whereas the power of incentives fails to increase and actually

declines with firm size.⁴ Finally, a high-powered incentive structure does not guarantee a high pay level. For example, the incentive structure of residual claimants is high-powered, but their pay level is low because of the small size of their businesses. The pay level of mid-level managers is equalized despite differences in talent and firm profits. Alignment between incentive structure and pay level occurs only with high-talent managers, who manage large businesses and also share their employers' profits.

In the model, the endogenous firm-size distribution mediates between the distribution of talent and the assignment of incentive contracts. This provides a new angle to analyse the effects of factors that influence firm-size distribution on the wage distribution. One such factor is TFP-enhancing technological progress, which can lead to disproportionate growth in firm size. Another factor is competition in product markets. In a market with heterogeneous firms, stiffer competition triggers production factors to be reallocated from smaller to larger firms. I show that an improvement in TFP-enhancing technology and greater product market competition result in an increase in the talent threshold of becoming a manager, the shrinkage of small-business owners and an expansion of high-talent managers who share firm profits, and an increase in the level of incentives offered to managers. These three effects jointly contribute to a highly-skewed wage distribution in favour of top talent.

This paper expands the seminal study of Lucas (1978) by embedding managerial firms into the price theory framework he develops, which provides a powerful tool to study the allocation of talent and its relation to industry characteristics. Earlier efforts along this line of research include Calvo and Wellisz (1979) and Rosen (1982), which model managerial firms with the span of control to account for the wage distribution within and across firms. More recently, Garicano and Rossi-Hansberg (2006) study the matching of individuals in the formation of hierarchic firms. Legros and Newman (2013) develop a price theory of firm boundary decisions and study the impact of technology and product demand on industry-wide re-organizations. As opposed to these studies that are focused on organizational forms, my study draws attention to the provision of managerial incentives. In the aspect of incentive provision, related to the current paper is Benabou and Tirole (2015), which embeds a multi-tasking moral hazard model into a Hotelling-like framework to analyse the impact of market competition and skill-biased technical change on the structure of compensation. Another related research is Powell (2015), which incorporates relational contracts into a model with heterogeneous firms to study the provision of managerial incentives and firm productivity.

The paper also contributes to the vast literature on managerial compensation. Based on the competitive assignment approach, a body of research attributes the disproportionately large amount of top-manager pay to firms' competition for managerial talent and the resulting assortative matching between managers and firms (e.g., Gabaix and Landier 2008; Tervio 2008). A number of recent studies (e.g., Edmans et al. 2009; Edmans and Gabaix

⁴See Gibbons and Murphy (1992), Schaefer (1998), Hall and Liebman (1998), and Baker and Hall (2004), among others, for the debate on the relationship between firm size and power of managerial incentives.

2011; Baranchuk et al. 2011; Bandiera et al. 2015) enhance this line of research by incorporating incentive problems into the competitive assignment framework. The current paper, in the spirit of Lucas (1978), departs from this literature by endogenising all firm characteristics without invoking competitive assignment in the labour market. Moreover, unlike prior studies that mostly focus on one type of incentive scheme for top managers, this paper analyzes a variety of incentive schemes, some of which are more relevant to managers in small businesses and middle managers of corporations. Finally, my model yields important implications regarding the employment of various types of managers, which is underexplored in the literature.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 establishes and characterizes the equilibrium. Section 4 applies the model to analyse the effects of technological progress and product market competition on managerial incentives and wage inequality. Section 5 discusses extensions that incorporate labour market mobility and barriers to market entry. Section 6 concludes. Proofs of the main theoretical results can be found in the appendix. Additional empirical results and proofs of technical results are relegated to an on-line appendix.

2 The Model

2.1 Economic Environment

Endowment. The population consists of a continuum of risk-neutral individuals, whose measure is normalized to one. Each individual is endowed with one unit of homogenous raw labour, but differs from others in the unidimensional managerial talent indexed by the scalar a . I assume that a is drawn from a continuous differential distribution $G(a)$ over $(0, \infty)$ with a well-defined probability density function $g(a)$.⁵ The economy is abundant in projects, all leading to the production of a homogeneous good. A project is owned by anonymous capitalists who are not explicitly modeled.

Demand and market. All individuals have the same preferences and demand the same homogeneous good. The product market is competitive, with firms as price-takers. The price of the good is normalized to one.

Production. Two inputs are indispensable for production. First, a firm's initial productivity is determined by the talent of the manager it employs. Second, a firm must hire production workers to generate output. For simplicity, I assume a Cobb-Douglas production form $f(a, n) = an^\beta$, with $\beta < 1$. Here, a is the total factor productivity, and n is the number of production workers.

Labour market. The market for production workers is perfectly mobile and competitive. A worker's supply of raw labour is perfectly inelastic. The wage of production workers, w ,

⁵I do not impose a restriction on the bounds of the talent distribution merely for expositional convenience. All the results will remain unchanged if the support of the talent distribution is bounded.

will be endogenous.

2.2 The Firm

A firm is defined by a manager, a number of production workers, and an employment relationship between a project owner and the manager. This definition of the firm abstracts from the complexity of team management and hierarchical structure. A firm can be regarded as an independent entity or as a plant or division of a multi-unit entity. This simplified definition does not seem inappropriate, as my focus is on incentive contracts and returns to factors.

For a given productivity a , a firm will choose its size, defined by the number of production workers it employs to maximize the following profit function:

$$\max_n \pi(n; a) = an^\beta - wn. \quad (1)$$

Thus far, the model is a simplified variant of Lucas (1978). I deviate from Lucas' model by allowing a manager to adjust her effort to improve firm productivity. Specifically, after exerting effort e , a manager increases a firm's productivity by a constant $\varphi > 1$ with probability e . This specification captures two features of a manager's function: 1) the local public good property of her service in the workplace, which shifts the firm's total factor productivity upward from a to φa ; 2) the uncertain aspect of her service, which makes the provision of managerial incentive an essential concern within the firm.

The expected profit of the firm that hires a manager with talent a is $e\pi(\varphi a) + (1 - e)\pi(a)$ wherein $\pi(\varphi a)$ and $\pi(a)$ are firm profits for realized firm productivity φa and a , respectively. Under this specification, managerial talent and effort are complementary. For simplicity, I assume that the cost function of e is $\frac{1}{2\xi}e^2$ with $\xi > 0$. Since e represents a probability measure, a sufficiently small ξ is assumed to ensure $e \in (0, 1)$. The main results of the model will not change with a general cost function as long as it is convex. Note that the cost of managerial effort is measured with final output rather than utility. Therefore, managerial effort can be regarded as an investment in human capital, or more generally, as resources needed to utilize managerial talent.

2.3 Contracting: A Limited-Liability Model

If there is no agency problem in the employment relationship, a manager with talent a will exert the first-best effort to maximize the firm's value – its expected profit net of the effort cost:

$$\max_e V(a) = e\pi(\varphi a) + (1 - e)\pi(a) - \frac{1}{2\xi}e^2. \quad (2)$$

However, the employment relationship between an owner and a manager is not frictionless. In this study, I focus on a particular type of friction – moral hazard due to unobservable managerial effort in conjunction with a limited-liability constraint that requires a minimum

Figure 1. Timeline of Contracting and Firm Formation

Meet	Contract	Hire	Produce	Pay
T_0	T_1	T_2	T_3	T_4
An anonymous owner meets an individual.	The owner offers pay contracts after observing the individual's ability.	Accepting the offer, individual hired as manager; Rejecting the offer, individual becomes worker.	Manager exerts effort; firm productivity realized; employment of workers adjusted	Firm profitability realized; manager and workers are paid.

level of compensation to employees in any state. The timing of the game is depicted in Figure 1. At time T_0 , a risk-neutral project owner meets one and only one individual who is entering the managerial labour market. At T_1 , the individual's talent is revealed to both parties, and the owner makes a take-it-or-leave-it offer. At T_2 , the individual decides whether to accept the offer. If the offer is accepted, the individual is hired as the manager of the owner's project; if the offer is rejected, the individual becomes a production worker. At T_3 , the employed manager exerts privately observable managerial effort to improve firm productivity. After observing the realized productivity, the firm makes production decisions and costlessly adjusts its employment of production workers. At T_4 , the output and firm profitability are realized, the contract is executed, and the manager and workers are paid. In this employment process, there are no information problems with respect to the manager's talent at the time of contracting. Inefficiency results only from moral hazard inside the firm.

In the above setup, I make the assumption that a manager who rejects an owner's offer becomes as a production worker so that all potential managers face the same outside option. Such an assumption is consistent with the static nature of the model: in this one-shot game, the managerial labour market opens only once, and neither a project owner nor an individual can go back to find another match when their contracting fails. Alternatively, this assumption might also be understood to define an extremely immobile labour market for managers, which contrasts with typical competitive assignment models that tend to the other extreme and assume a perfectly mobile managerial labour market. Of course, reality lies between these two assumptions. I assume an immobile managerial labour market to highlight the incentive problems within firms. In Section 5, I will relax this assumption to incorporate a more flexible labour market.

Formally, an owner will offer a manager with talent a a wage profile $\{s(a), b(a)\}$, where $s(a)$ is a salary invariant with firm profits and $b(a)$ is a bonus paid to the manager when the firm's productivity is improved to φa .

The owner faces the following constrained-optimization problem:

$$\max_{b(\cdot), s(\cdot)} V(a) = e[\pi(\varphi a) - b(a)] + (1 - e)\pi(a) - s(a) \quad (3)$$

subject to

$$\begin{aligned}
 (PC) : eb(a) - \frac{1}{2\xi}e^2 + s(a) &\geq w, \\
 (IC) : e \in \arg \max_{e'} e'b(a) - \frac{1}{2\xi}e'^2 + s(a), \\
 (WC) : \min\{b(a) + s(a), s(a)\} &\geq \underline{w}.
 \end{aligned}$$

Here, $\pi(\cdot)$ is a firm's ex post profit function, as defined in (1). *PC* is the participation constraint, meaning that the net return to an employed manager should be no less than her outside option as a production worker. *IC* is the incentive compatibility constraint. *WC* is the limited-liability or wealth constraint: the owner cannot pay the manager less than \underline{w} in any state. I interpret \underline{w} as a legal or institutional requirement that is independent of a manager's talent. In this economy without capital, the only wealth to meet the limited-liability constraint is one's realized wage income. Given that the wage of production workers is w , a meaningful wealth constraint is $\underline{w} \leq w$, an assumption that will be made throughout the rest of this paper. Note that \underline{w} can take a negative value because what it indicates is a firm's ability to punish managerial failure.

3 Equilibrium Analysis

The equilibrium of the economy is defined by the following conditions: 1) All individuals optimally choose their employment status and have no incentive to deviate from their current employment; 2) All the incentive contracts are optimally designed and managers exert optimal efforts accordingly; 3) Firms make optimal production decisions regarding output and the employment of production workers; 4) A firm is active if and only if it receives a non-negative expected payoff; and 5) Both the labour market and the product market clear.

3.1 Optimal Decisions inside the Firm

3.1.1 Optimal Production

Solving the profit-maximizing problem (1), the optimal size – measured by the number of production workers – of a firm with productivity a is

$$n(a) = \left(\frac{\beta a}{w}\right)^{\frac{1}{1-\beta}}. \quad (4)$$

A firm's employment increases in its productivity and decreases in the market wage. $\beta < 1$ captures the degree of scale economies and governs firm size. When $\beta \rightarrow 1$, the production function features constant returns to scale and the most productive firm will take over the whole market. For notational convenience, let $\sigma = \frac{1}{1-\beta}$, which has a similar interpretation as β but is greater than one.

Using (4), the profit function (1) can be written as

$$\pi(a) = \frac{1}{\sigma - 1} wn(a). \quad (5)$$

The contribution of productivity to a firm's profit is completely absorbed by the firm's size. The relative profit and size of any two firms with different levels of productivity can then be explicitly expressed as:

$$\frac{\pi(a_i)}{\pi(a_j)} = \frac{n(a_i)}{n(a_j)} = \left(\frac{a_i}{a_j}\right)^\sigma \text{ for all } i, j. \quad (6)$$

3.1.2 Optimal Incentive Contracts

As a benchmark, the first-best effort, denoted as e^{FB} , that solves the problem (2) is:

$$\begin{aligned} e^{FB}(a) &= \xi \Phi \pi(a) \\ &= \frac{\xi \Phi}{\sigma - 1} wn(a), \end{aligned} \quad (7)$$

where $\Phi = \varphi^{\frac{1}{1-\beta}} - 1$ measures the improvement in productivity achieved by successful management.

I now solve the constrained-optimization problem (3). The solution will depend on whether or not the participation constraint and the wealth constraint are binding. From the perspective of the owner, the wealth constraint indicates her limitation to punish the manager. This limitation forces the owner to "excessively" reward the manager in a good state to elicit managerial effort. When the participation constraint is slack, it is optimal for the owner to push down the limit of punishment as much as possible to minimize the cost of providing incentives. When the wealth constraint is slack, an owner still has scope to punish a shirking manager, and there is no need to offer rent to the manager. This leads to the following result:

Lemma 1 *At least one of the two constraints — the wealth constraint or the participation constraint — must bind at optimum.*

Proof. See Appendix A1. ■

When the wealth constraint is not binding, the owner can implement the first-best effort by "selling the store" to the manager even if her effort is not observable.

Lemma 2 *The first-best effort is implemented $e^{FB}(a) = \xi \Phi \pi(a)$ if and only if $\Phi \pi(a) < \sqrt{\frac{2(w-w)}{\xi}}$.*

Proof. See Appendix A2. ■

The above lemma states that the surplus of the firm is so small that the wealth constraint is slack and the manager can buy the "store" from the owner. Able to make a take-it-or-leave-it offer, the owner sets the selling price of "the store" to extract the entire surplus.

The result that the agency problem between risk-neutral parties can be fully resolved when the wealth constraint is slack is well-known in contract theory. Lemma 2 explicates the condition that justifies the feasibility of such a first-best solution: the surplus created by the manager must be bounded by a monotone function of $w - \underline{w}$, which measures the difference between the manager's outside option and the limit of wealth. Intuitively, when the manager has a greater outside option, the owner must "sell the store" at a higher price to induce participation; selling a larger "store" is thus feasible. When the limit of wealth is further relaxed (\underline{w} is smaller), an owner's ability to extract surplus by selling her "store" is less constrained. Therefore, the feasibility of efficient contracts depends on the tension between the participation constraint and the wealth constraint. This tension, endogenous in market equilibrium, will determine the stratification of agents with different ability into two types of incentive contracts – residual claim and contingent pay.

When the firm surplus is sufficiently large, the wealth constraint binds and the manager's incentive is distorted.

Lemma 3 *Suppose the wealth constraint is binding.*

1) *When the participation constraint is binding (indicated by BP), the optimal incentive contract takes the form: $\{s(a) = \underline{w}, b^{BP}(a) = \sqrt{\frac{2(w-\underline{w})}{\xi}}\}$; managers with different ability exert the same amount of effort $e^{BP}(a) = \sqrt{2\xi(w-\underline{w})}$.⁶*

2) *When the participation constraint is slack (indicated by RP), the optimal contract is $\{s(a) = \underline{w}, b^{RP}(a) = \frac{1}{2}\Phi\pi(a)\}$; managers' effort varies with firm profitability and thus managerial ability: $e^{RP}(a) = \frac{\xi}{2}\Phi\pi(a)$.*

Proof. See Appendix A3. ■

Intuitively, the owner intends to use a performance-pay scheme to elicit managerial effort by rewarding good management (firm productivity φa) and punishing bad management (firm productivity a). High pay for good management induces a larger managerial effort but leaves the manager a positive rent over her outside option. When the value of managerial effort is not sufficiently large, a firm aims to minimize the limited-liability rent and pushes down the payment until the participation constraint becomes binding. The manager then will only exert a constant amount of effort independent of firm profit and her ability; the owner pays a bonus that is tied to the manager's outside option and invariant with her ability. When the managerial effort is sufficiently valuable, the owner aims to maximize feasible managerial effort; the participation constraint is no longer binding. Optimal incentive design requires that the pay differential between the two states be tied to the firm's profit. Accordingly, managerial effort varies with managerial ability.

The novelty of Lemma 3 lies in its emphasis on the optimal contract design in a distri-

⁶Note that in this solution, $s(a)$ takes a negative value if $\underline{w} < 0$. However, $b(a)$ is always positive as a non-positive $b(a)$ requires $w \leq \underline{w}$, in which case the participation constraint will not be binding and this type of contract will not exist.

butional context. In a traditional model of moral hazard due to limited liability,⁷ the focus is within a specific principal-agent relationship, and optimal contractual forms are studied in isolation. In the current model with heterogeneous agents competing in the market, a variety of optimal contractual forms emerge for managers across different ability segments. This insight forms the basis of the sorting and distributional results in the market equilibrium that will be presented later.

I will refer to the contract with a binding participation constraint as a "one-step-bonus," meaning an constant increment of pay in the successful state relative to the pay in the failure state, i.e., similar to a step function. For expositional convenience, I will also call this type of contract a "bonus contract". I will refer to the contract with a slack participation constraint as a "rent-sharing contract" because it is the only type of contract that transfers part of rent to managers.

3.2 Partial Equilibrium: Sorting of Firms to Incentive Contracts

The equilibrium will ultimately be characterized by individuals sorted into various incentive structures and a corresponding wage distribution in the economy. These two features are intermediated by an endogenous firm-size distribution. To elucidate this intermediating mechanism, I solve for a partial equilibrium of the model, assuming the wage of production workers as given.

In the model, firms offer optimal contracts to managers. Therefore, a firm selects the contract that maximises its value functions as specified in (2) and (3). Substituting into these value functions both the optimal managerial effort and pay as specified in Lemmas 2 and 3 and the relationship between firm profit and firm size (5), the value of a firm under three types of contract is as follows:

$$\begin{aligned}
 V^{sell}[n(a)] &= \frac{\xi}{2} \left(\frac{\Phi}{\sigma - 1} \right)^2 w^2 n(a)^2 + \frac{wn(a)}{\sigma - 1} - w; \\
 V^{bonus}[n(a)] &= [\sqrt{2\xi(w - \underline{w})\Phi} + 1] \frac{wn(a)}{\sigma - 1} - (2w - \underline{w}); \\
 V^{rent}[n(a)] &= \frac{\xi}{4} \left(\frac{\Phi}{\sigma - 1} \right)^2 w^2 n(a)^2 + \frac{wn(a)}{\sigma - 1} - \underline{w}.
 \end{aligned}$$

The superscript "sell" indicates the "sell-the-store" contract; "bonus" indicates the bonus contract; and "rent" indicates the rent-sharing contract. Here, I write a firm's value in terms of $n(a)$, which is the size of the firm whose manager converts her ability, a , to firm productivity but fails to improve firm productivity despite her effort. Note that the model setup in Section 2.3 implies that a firm is not yet established at the contracting stage. To be consistent with this setup, $n(a)$ should be viewed as the correctly-anticipated size of a firm whose realized productivity remains at its initial level, a .

⁷See, for instance, Sappington (1983), Brander and Spencer (1989), Innes (1990), Laffont and Martimort (2002), and Jewitt et al. (2008).

Equating $V^{sell}[n(a)] = V^{bonus}[n(a)]$ and $V^{bonus}[n(a)] = V^{rent}[n(a)]$ defines two threshold values of firm size:

$$n^* = \frac{\sigma - 1}{w\Phi} \sqrt{\frac{2(w - \underline{w})}{\xi}}, \quad (8)$$

$$n^{**} = \frac{2(\sigma - 1)}{w\Phi} \sqrt{\frac{2(w - \underline{w})}{\xi}}. \quad (9)$$

A firm of size n^* is indifferent between offering a "sell-the-store" contract and a bonus contract; a firm of size n^{**} is indifferent between offering a bonus contract and a rent-sharing contract. Moreover, $e^{BP} = e^{FB}$ at n^* and $e^{RP} = e^{BP}$ at n^{**} . Note that n^{**} is double the value of n^* because of the quadratic form of the cost function for managerial effort.

Proposition 1 *Define a pair of threshold values $\{n^*, n^{**}\}$ as in (8) and (9). A firm will "sell the store" to the manager when its size is small: $n(a) < n^*$; a firm will adopt a one-step-bonus incentive contract when its size is medium: $n(a) \in (n^*, n^{**})$; and a firm will adopt a rent-sharing incentive contract when its size is large: $n(a) > n^{**}$.*

Proof. See Appendix A4. ■

This proposition compares three types of incentive contracts across firms of different initial size. For small firms, the optimal contract is to transfer ownership to the manager who can afford the firm with her future income. The manager becomes a residual claimant, and the owner sets the price to extract the entire surplus. A medium-sized firm is too large to validate a transfer-ownership contract, but is too small for a rent-sharing contract to be worthwhile. In particular, given the modest size of the surplus to be shared, using a rent-sharing contract to induce participation would require a firm to give up a large share of the surplus. Such a firm prefers a bonus contract, which only requires compensating the manager for her outside option and the cost of effort. By contrast, a large firm optimally chooses a rent-sharing contract because the share of surplus that must be given up to ensure participation and elicit high effort is small.

3.3 Market Equilibrium

Two important variables are yet to be determined: the wage of production workers w and the cutoff value of talent \underline{a} that divides the population into two classes: managers and production workers. The equilibrium values of these two variables will be determined by free entry of firms and market clearing.

Free entry implies that a firm will enter the market if and only if its expected profit is non-negative. Therefore, the marginal firm, which "sells the store" to a manager, must break even:

$$V^{sell}(\underline{a}, w) = \frac{\xi}{2} \left(\frac{\Phi}{\sigma - 1} \right)^2 w^2 \underline{n}^2 + \frac{w\underline{n}}{\sigma - 1} - w = 0, \quad (10)$$

where $\underline{n} \equiv n(\underline{a})$ denotes the size of the marginal firm whose productivity is the minimum among all surviving firms.

Production workers in any firms receive the same wage which equals their marginal product. Evaluate Equation (4) for the marginal firm, and the economy-wide wage can be written as

$$w = \frac{\sigma - 1}{\sigma} \underline{a} \underline{n}^{-\frac{1}{\sigma}}. \quad (11)$$

Clearing the labour market requires that every individual in the economy be employed, either as a production worker or a manager:

$$\int_{\underline{a}}^{\infty} \{[e(a)n(\varphi a) + [1 - e(a)]n(a)]g(a)da = \int_0^{\underline{a}} g(a)da. \quad (12)$$

The left-hand side is the demand for raw labour, aggregating the number of production workers for each firm across the entire range of possible realized productivity. The right-hand side is the supply of raw labour, consisting of individuals whose talent below the threshold for being a manager. When the labour market clears, the product market clears as well.

From the free-entry condition (10), the minimum firm size is obtained as:

$$\underline{n} \equiv n(\underline{a}) = \frac{\sigma - 1}{w} \frac{-1 + \sqrt{1 + 2w\xi\Phi^2}}{\xi\Phi^2}. \quad (13)$$

As $n(a)$ is a strictly increasing function in a , I denote a triple threshold of managerial talent as $\{\underline{a}, a^*, a^{**}\}$, corresponding to the triple threshold of firm size $\{\underline{n}, n^*, n^{**}\}$ defined, respectively, in (13), (8), and (9). The relationship between relative firm size and managerial talent (6) yields

$$\frac{a^*}{\underline{a}} = \left[\frac{n^*}{\underline{n}}\right]^{\frac{1}{\sigma}} \quad \text{and} \quad \frac{a^{**}}{a^*} = \left[\frac{n^{**}}{n^*}\right]^{\frac{1}{\sigma}} = 2^{\frac{1}{\sigma}}. \quad (14)$$

With these building blocks, a triple of $\{\underline{a}, \underline{n}, w\}$ is pinned down by (10), (11), and (12), and the economy-wide equilibrium is fully determined.

Lemma 4 *There always exists a unique pair (\underline{a}, w) such that both the market clearing condition and the free entry condition are satisfied.*

Proof. See Appendix A5. ■

Once the equilibrium (\underline{a}, w) is established, the other threshold values a^* and a^{**} can be obtained immediately from (14). Accordingly, the equilibrium distribution of optimal incentive contracts, managerial effort, firm size, and wages over all individuals can be computed.

3.4 Characterization of the Equilibrium

3.4.1 Sorting of Talent to Incentive Contracts

Combining Proposition 1 and Lemma 4 leads to the equilibrium sorting pattern of incentive contracts on the basis of managerial talent.

Proposition 2 *In equilibrium, the population is sorted into four groups: 1) least-talent individuals ($a < \underline{a}$) are production workers; 2) low-talent individuals ($\underline{a} \leq a < a^*$) are residual claimants; 3) medium-talent individuals ($a^* \leq a < a^{**}$) are salaried managers paid a bonus contract; and 4) high-talent individuals ($a \geq a^{**}$) are salaried managers paid a rent-sharing contract.*

Proposition 2 presents the central result of this paper. The population is stratified into three types of employment status: production workers, small business owners, and salaried managers, corresponding to three basic types of incentive contracts: fixed salary, residual claim, and contingent pay. The division between production workers and managers is due to the scarcity of managerial jobs: minimum firm productivity (and thus minimum managerial talent) is required to survive market competition. Below this cut-off value, individuals, although of various talent levels, become production workers. Above the threshold talent for being a manager, the constraint of limited liability generates a size effect that allocates talent to various incentive structures. The low-talent managers create a small surplus that can be financed by their certain income. However, provided that the owners can extract the entire surplus, it is costless to fully align managerial incentives by transferring ownership to the manager. This contractual arrangement, which is analogous to a market transaction, simultaneously achieves efficiency and low cost. When ownership transfer is not feasible, providing incentives becomes costly. The value of effort elicited from a medium-talent manager is not sufficient to outweigh the cost of providing a high level of incentive. The owner is not willing to sacrifice rent and thus uses a contingent-pay structure tied to the manager's outside option to elicit managerial effort. This type of contract is relatively low cost and has modest benefits. By contrast, a high-talent manager creates a sufficient surplus such that the owner is willing to offer part of the rent created in the employment relationship. Such a rent-sharing contract incurs a high cost but has a large benefit.

From the perspective of managers, assigning incentive contracts on the basis of talent reflects managerial ability to protect the surplus they create. Low-talent managers create small surplus and subject themselves to the "exploitation" of owners, who use the "sell-the-store" contract to induce the first-best effort and then extract the entire surplus. Medium-talent managers create larger surplus, which makes them less subject to owners' "exploitation" in the sense that owners can only extract the surplus created by a second-best level of effort. Given that their pay is independent of their ability, these managers exert a constant effort that is invariant with their ability and firm performance. They are thus often described as

"working like bureaucrats." High-talent managers are able to retain part of the surplus they create because of owners' intention to elicit their highly valuable effort. Therefore, they are viewed as "managerial entrepreneurs" who work hard for the prosperity of the firm.

Empirically, residual claimants in this model can be interpreted as small business owners and self-employed workers, including sole proprietors, home-based business owners, and franchisees. These individuals own and run a wide range of businesses such as grocery stores, small farms, hair salons, laundry shops, fast-food restaurants, and simple professional and managerial services. According to the U.S. Census Bureau, very small businesses with fewer than 20 employees comprised approximately 90 per cent of employer firms and 18 per cent of private sector payroll during the 2000s in the US. Despite declining self-employment in the US, the number of self-employed remained more than 15 million in 2010, accounting for approximately 10 per cent of total employment. The education and skill levels of small business owners and the self-employed are only slightly higher than those of an average wage worker.⁸

The empirical counterparts of salaried managers who receive a rent-sharing contract can be understood as senior managers in large companies, who are commonly offered pay schemes that tie their wages to firm performance. Along the managerial skill and responsibility ladder, these managers are at the highest levels; within the managerial occupation, they are classified as top executives or chief general managers. It is more complex to map salaried managers who receive a bonus contract to an empirical counterparts. The present theory does not clearly draw boundaries among firms, so these managers might be viewed as general managers in medium-sized firms or as division and plant managers in large firms. In this paper, I provide two proxies for these two types of managers without distinguishing between them. One proxy consists of individuals in a managerial occupation whose jobs require medium-range skills and responsibilities, whereas the other consists of individuals excluded from classification as top executives or chief general managers within the managerial occupation.

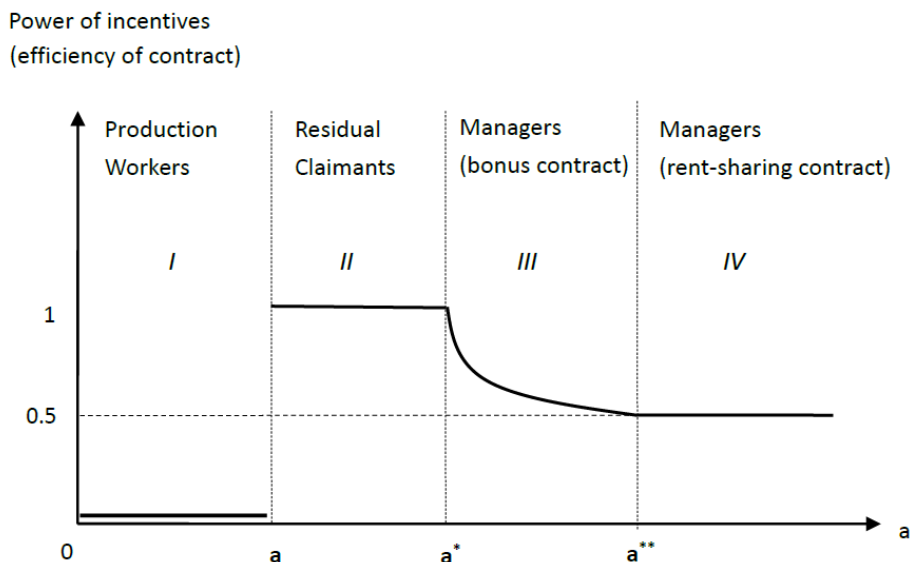
3.4.2 Power of Incentive Contracts

To further contrast the above different incentive structures across individuals, I measure the power of incentives by calculating the sensitivity of pay to firm profits, defined as the ratio of the covariance between managerial pay and firm profit to the variance of firm profit. This measure corresponds to the coefficient of regressing an individual's realized pay on firm profit for all individuals at the same skill level. It also coincides with a measure of efficiency defined as the ratio between a manager's realized effort and the first-best effort that the manager would undertake in the absence of contractual frictions.

Proposition 3 *In equilibrium, the power of incentive structure is non-monotone in talent;*

⁸Based on U.S. Bureau of Labor Statistics data in 2010, the proportion of individuals who have a college degree or higher was found to be approximately 41 percent among the unincorporated self-employed, 56 percent among the incorporated self-employed, and 46 percent among wage and salary workers. The 2007 Survey of Business Owners conducted by the U.S. Census Bureau showed that approximately 51 percent of business owners - the majority of whom are small business owners - have a college degree or higher.

Figure 2: Talent Distribution and the Power of Optimal Incentive Contracts



among managers, the power of incentive structure is weakly decreasing in talent, whereas the size of incentives is weakly increasing in talent.

Proof. See Appendix A6. ■

Figure 2 illustrates the above result. Production workers receive a flat wage, as they supply their raw labour inelastically. The power of their incentive structure is zero. Above this threshold value, individuals become managers, whose pay is contingent on firm profit. The least-talented managers are residual claimants. Their pay structure is efficient and the power of their incentives is extremely high. Individuals with greater talent become salaried managers and cannot fully tie their pay to firm profit. Among this group, medium-talent managers are rewarded with a bonus contract. Given the constant pay differential for these managers, the power of their incentive structure decreases in talent up to a threshold value such that the owner is willing to share rent (region III in Figure 2). High-talent managers receive the rent-sharing contract, and the power of their incentives remains constant – half in the model because of the quadratic cost function of managerial effort (region IV).

It may seem surprising that the power of the rent-sharing contract is lower than that of the bonus contract, as the latter is adopted to avoid giving up rent at the cost of muting incentives. In a typical partial-equilibrium model of contracting with limited liability, a bonus contract is regarded as less-powered than a rent-sharing contract because the comparison is within the same manager in the same employment relationship. The current model does not contradict this result. In region III of Figure 2, a rent-sharing contract that ensures participation must offer a larger share of rent than the same type of contract in region IV and is more powerful than the bonus contract. In region IV, if the firm continues to use the bonus contract, the power of incentive will be lower than that of the rent-sharing contract.

The result in Proposition 3 helps to resolve two views in the debate about managerial compensation. One is that incentives are inefficiently low in large firms (e.g., Bebchuk and Fried 2004), supported by the negative relationship between dollar-to-dollar incentives and firm size (e.g., Baker and Hall 2004). The other is that the size of incentives is greater in larger firms, supported by the positive relationship between wealth-performance sensitivity and firm size (e.g., Becker 2006).⁹

3.4.3 Wage Distribution

The sorting pattern described in Proposition 2 also determines the relationship between managerial ability and pay level. I will characterize a manager's wage earnings in terms of the expected value of pay. At the individual level, the expected pay reflects one's life-time wage income, consistent with the static nature of the model. At the cross-sectional level, the expected pay would reflect the average wage of individuals with the same ability, which is the focus of the empirical examination in this paper. The function of expected pay $W(a)$ – referred to as a wage function for simplicity – can be written in terms of a firm's initial size $n(a)$, which in turn is a monotone function of talent:

$$W^{worker}(a) = w \quad \text{for } a \in (0, \underline{a}) ; \quad (15)$$

$$W^{residual}(a) = w + \frac{\xi}{2} \left(\frac{\Phi}{\sigma - 1} \right)^2 w^2 n(a)^2 \quad \text{for } a \in [\underline{a}, a^*]; \quad (16)$$

$$W^{bonus}(a) = 2w - \underline{w} \quad \text{for } a \in [a^*, a^{**}];$$

$$W^{rent}(a) = \underline{w} + \frac{\xi}{4} \left(\frac{\Phi}{\sigma - 1} \right)^2 w^2 n(a)^2 \quad \text{for } a \in [a^{**}, \infty).$$

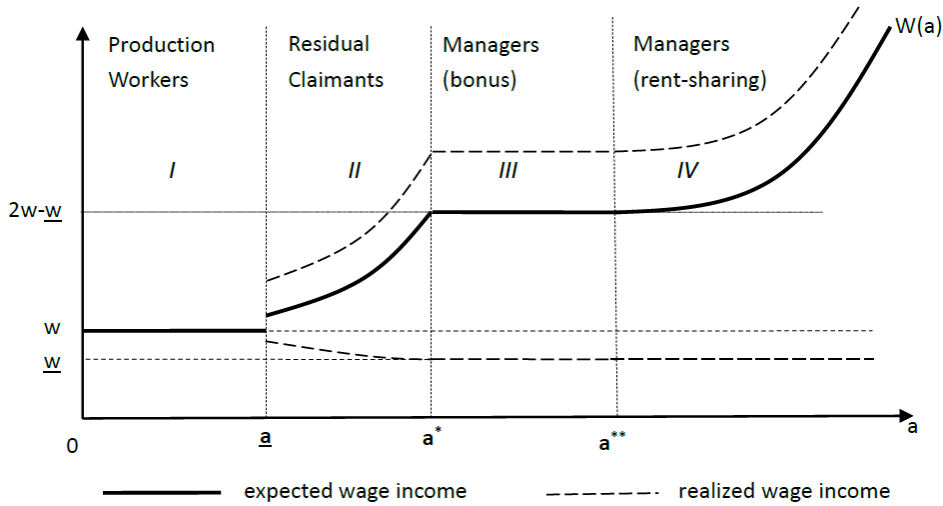
The superscripts *worker*, *residual*, *bonus* and *rent* indicate, respectively, the four types of individuals: production workers, residual claimants, managers paid a bonus contract, and managers who share rent. The distribution of incentive structures can be characterized by the proportion of each managerial group defined by incentive contracts, denoted as follows:

$$\underline{\theta} = \frac{\int_{\underline{a}}^{a^*} g(a) da}{\int_{\underline{a}}^{\infty} g(a) da}; \quad \theta^* = \frac{\int_{a^*}^{a^{**}} g(a) da}{\int_{\underline{a}}^{\infty} g(a) da}; \quad \theta^{**} = \frac{\int_{a^{**}}^{\infty} g(a) da}{\int_{\underline{a}}^{\infty} g(a) da}. \quad (17)$$

Figure 3 depicts the wage curve against managerial talent, divided into four regions. First, production workers simply earn the economy-wide wage determined by the supply and demand in the raw labour market. Second, a residual claimant earns her outside option as a production worker and a variable part that compensates for her effort. Third, a salaried manager who receives a bonus contract earns a constant wage to compensate for her outside option and the fixed effort that is determined by the difference between her outside option

⁹Certainly, CEO pay is a complicated issue. The empirical relationship between managerial incentives and firm performance varies under different incentive measures and in different samples. Our model is qualitatively consistent with some broad patterns of managerial compensation.

Figure 3: Sorting of Talent and the Wage Distribution



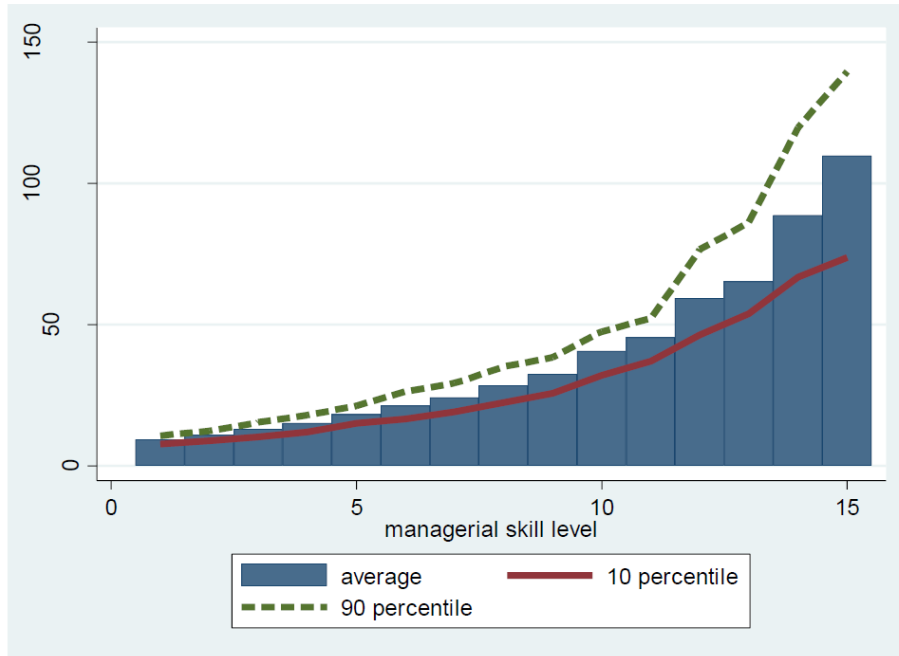
and the wealth constraint. Thus, the wage of these salaried managers varies closely with the wage of production workers and is independent of their own talent and the size of their employers. Finally, the wages of salaried managers who receive a rent-sharing contract are amplified by firm profits and increase rapidly in their talent. The wage function of these managers $W^{rent}(a)$ is proportional to $n(a)^2$, which is convex in a .

To demonstrate the variation in pay both within each managerial group and across different manager groups, Figure 3 also depicts the distribution of the realized wage income (the dashed lines). In general, the pay difference between the good and bad states for a fixed ability level, i.e., the size of bonus, weakly increases with managerial ability. However, the variation in bonus across ability levels is not monotonic in managerial ability. Among both residual claimants and rent-sharing managers, the size of bonus increases with ability, whereas among salaried managers paid a bonus, the size of bonus does not vary at all.

3.5 Empirical Evidence

I examine the model empirically by exploring the National Compensation Survey (NCS) data covering the 2006-2010 period, collected by the U.S. Bureau of Labour Statistics. The publicly available data of these large-scale surveys contain pay information at the narrow occupational level in all major industries and across firms of different size. Although the data set does not identify business owners and does not include the self-employed, it has the advantage of clearly defining managerial occupations and of providing a measure of managerial ability based on the skill and responsibility required for an occupation. For the empirical counterparts to production workers in the model, I use occupations that are classified as "production workers" or related to "clerical/administrative/secretary" jobs. In the empirical description, I use "workers" instead of "production workers" to represent all these types of workers (as

Figure 4. Distribution of Hourly Wages across Managerial Levels



opposed to managers).

I first examine the model's implications for wage inequality between managers and workers. According to the model, managers as a whole earn a wage premium over workers as the result of 1) the selection effect that allows managers to utilize their managerial ability, 2) incentive contracts used to elicit managerial effort, and 3) firm size which amplifies returns to managerial ability. Table 1 presents evidence consistent with these three drivers of the managerial wage premium. First, during the sample period, the average hourly wage of a manager is approximately \$38, which is more than double that of a worker.¹⁰ Second, within the broad class of managers, those who receive incentive pay earn significantly more than those who earn time-pay salaries, whereas such a difference is negligible among workers. The third set of columns presents the hourly wages of workers and managers in firms of different size. Consistent with the model's predictions, managers' wages increase significantly with firm size, whereas the wages of workers in firms with fewer than 500 employees vary little with firm size.¹¹

My second examination concerns the wage distribution across individuals with heterogeneous managerial ability. Figure 4 plots the hourly wages against the "managerial level"

¹⁰The managerial wages in the NCS data do not include option and other equity-based forms of pay that are typically part of compensation packages for top managers in large firms. Therefore, the average managerial pay presented here should be regarded as a lower bound.

¹¹One feature of the data that is inconsistent with the theory developed in this paper is that workers employed by firms with more than 500 employees appear to earn a wage premium compared with their counterparts in smaller firms. This may indicate special compensation practices in very large firms, such as unionisation of workers.

Table 1. Hourly Wages of Production and Clerical Workers and Managers by Pay Structure and Firm Size

	All Workers	incentive pay		firm size (#employees)			
	full-time, industry	by time	by incentive	1-49	50-99	100-499	>500
production workers							
average wage (hourly)	15.79	15.77	16.03	14.22	14.79	15.03	19.62
standard deviation	(0.62)	(0.65)	(1.13)	(0.74)	(1.40)	(0.74)	(0.96)
clerical and administrative workers							
average wage (hourly)	15.78	15.77	15.94	15.21	15.68	15.72	17.07
standard deviation	(1.11)	(1.10)	(1.37)	(1.10)	(1.23)	(1.01)	(1.48)
managers							
average wage (hourly)	37.72	37.35	43.09	32.19	36.91	38.98	42.99
standard deviation	(1.98)	(2.32)	(3.98)	(1.34)	(1.44)	(3.17)	(3.06)

Notes: The data are the average hourly wage for each broad occupation category from 2006 to 2010 in the U.S. The standard deviations are calculated based on yearly observations. Data source: U.S. Bureau of Labor Statistics.

Table 2. The Effects of Managerial Ability on Wages among Different Skill Groups

Dependent variable	all skill levels (Levels 1=15)		low skill levels (Levels 1-5)		midde skill levels (Levels 6-10)		top skill levels (Levels 11-15)	
	ln (hourly wage)		ln(hourly wages)		ln(hourly wages)		ln(hourly wages)	
skill level	0.145*** (0.006)	0.146*** (0.006)	0.135*** (0.013)	0.134*** (0.013)	0.086** (0.037)	0.094** (0.037)	0.440** (0.192)	0.454** (0.198)
occupation fixed effects	yes	yes	yes	yes	yes	yes	yes	yes
year fixed effects	no	yes	no	yes	no	yes	no	yes
#observations	11981	11981	5758	5758	4905	4905	1318	1318
R-squared	0.954	0.958	0.898	0.907	0.858	0.871	0.848	0.868

Notes: The independent variable is the logarithm of the average hourly wage at the occupation_year level from 2006 to 2010. Skill levels are classified by the U.S. Occupational Compensation Survey, ranging from 1 to 15 on the basis of skill and responsibility required for managerial jobs. Standard errors are clustered at the occupation level. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Data source: U.S. Bureau of Labor Statistics.

defined in the NCS data. Two features are consistent with the theoretical wage distribution depicted in Figure 3. First, there is a convex relationship between wages and managerial levels. Second, as a measure of pay dispersion, the difference between the 10th and 90th percentile wages at the same managerial level widens as the managerial level rises.

Whereas the convex relationship between wages and managerial levels can also be explained by other theories, such as the competitive assignment theory, the current theory makes a unique prediction regarding medium-talent individuals. As discussed above, the inter-person wage variation among this group of managers should be smaller than the same variation in the other two managerial groups, because the size of the bonuses they receive is invariant with ability and firm profitability. Table 2 presents the results from regressing individuals' wages (in logarithm) on their managerial levels for each of the three managerial groups. The results show that among medium-talent managers, a one-level increase in managerial ability increases wages by approximately 9%, whereas the same increase in the managerial level raises wages by 45% among top managers and by 13% among low-talent managers. This non-monotonic result provides suggestive evidence in support of the current theory, indicating that a convex pay-skill relationship results not only from the assortative matching between firms and managers but also from the rent-sharing incentive structure.

4 Applications

It is well-documented that wage inequality between unskilled and skilled workers – particularly the gap between the top and average wage income earners – has widened in the past several decades (e.g., Piketty and Saez 2003, 2006; Acemoglu and Autor 2012). Economists have offered various explanations for such rising inequalities, including skill-biased technology progress, globalization, labour market competition, and top managers’ rent-seeking. The current model provides a new angle that addresses the issue of wage inequality through channels involving the firm-size distribution. In this section, I apply the model to analyze the effects of two factors that may affect the firm-size distribution: 1) Hicks-neutral technological progress and 2) increased competition in the product market.

4.1 The Impact of Hicks-neutral Technological Progress

In the neoclassical framework, a Hicks-neutral technological change that improves firms’ total factor productivity, i.e., TFP-enhancing technological progress, does not affect the returns of skilled workers relative to unskilled workers. In the present model, however, TFP-enhancing technological progress will affect the division between production workers and managers as well as the composition of incentive contracts for managers. This effect will change the relative returns to skilled and unskilled workers and wage inequality within the managerial class – one important class of skilled workers.

To incorporate technological progress into the model, I amend the production function in the baseline model: the output of a firm with productivity a is $f(a, t) = tan(a, t)^\beta$. The new component $t > 1$ indicates an economy-wide technology that is multiplicable by a firm’s idiosyncratic productivity a . An increase in t implies an improvement in the TFP of all firms.¹² With this new production function, the profit of a firm with productivity a is:

$$\pi(a, t) = \frac{1}{\sigma - 1} w(t) n(a, t),$$

which is analogous to (1). The contribution of t to a firm’s profit is completely absorbed by the wage of production workers and firm size. Therefore, the free-entry and market-clearing conditions (10) and (12) in Section 3 remain unchanged. The wage function (11), however, must incorporate the economy-wide technology factor:

$$w = \frac{\sigma - 1}{\sigma} \underline{tan}^{-\frac{1}{\sigma}}, \quad (18)$$

where \underline{a} and \underline{n} are, respectively, the productivity and size of the marginal firm, as previously defined.

In a traditional model with homogeneous firms, a technological improvement from t to t'

¹²The single factor production function in this paper can be thought of as a reduced form for a constant returns to scale production function with multiple factors.

will increase the marginal product of raw labour, leading to a rise in the wage of production workers. In this model with heterogeneous firms, such a technological improvement increases a firm's demand for production workers by a factor of $(\frac{t'}{t})^\sigma$. Therefore, upon the technological shock, the labour demand of more-productive firms responds substantially more than that of less-productive firms. To restore the equilibrium in the labour market, the least-productive firms ought to exit to release additional production workers to the labour market.

Lemma 5 (*Positive Selection Effect*) *A Hicks-neutral technological change that improves firms' total factor productivity (t) increases the wage of production workers (w) and the threshold talent for becoming a manager (\underline{a}).*

Proof. See Online Appendix C1.¹³ ■

The essence of this lemma is that technological progress intensifies competition for scarce resources – production workers in this model – in favour of firms whose managers are able to use resources more efficiently. The market then adjusts the price of scarce resources such that the least-productive firms cannot afford it. In other words, the competition for production workers drives up the wage of production workers and drives out the least-productive firms. In consequence, the least-talented managers must downgrade their occupation. A similar positive selection effect can be obtained in a Lucas' (1978) type of model without managerial effort. In the present model, this selection effect is amplified by managerial effort. In addition to the impact on the margin between production workers and managers, technological progress leads to adjustments of incentive contracts and changes in the margins between different types of managers. To highlight the essential mechanisms, I make the following assumption with regard to the distribution of talent.

Assumption 1 *The distribution of managerial talent among the whole population is Pareto: $G(a) = 1 - a^{-\lambda}$ over $(0, \infty)$.*¹⁴

The Pareto distribution is widely used to fit the empirical studies of income distribution and firm-size distribution.¹⁵ This distribution has an analytical advantage because a Pareto distribution truncated from below remains a Pareto distribution with a changed lower bound. In the form of Pareto distribution in Assumption 1, the single shape parameter λ measures the inequality of the variable of interest. A larger λ implies a smaller level of inequality.

¹³Despite the fairly straightforward intuition behind this lemma, the proof is rather tedious because of the general equilibrium effect on the adjustment on the contractual margin. Therefore, I relegate the proof to the on-line appendix (Appendix C).

¹⁴This Pareto distribution implies $a \geq 1$. I write the support of the distribution as $(0, \infty)$ to be consistent with the notation in Section 2. The boundary values of the support do not matter for the analysis.

¹⁵The Pareto distribution has been used to characterize income distribution and inequality (Arnold 1983) and the distribution of firm size (Axtell 2001). A large body of recent empirical literature uses the Pareto distribution to interpolate the top wage distribution (see Atkinson et al 2011 for a survey).

Owing to Assumption 1, the share of each manager type among the managerial class, defined in (17), can be written solely in terms of the boundary values of firm size as follows:

$$\begin{aligned}\underline{\theta} &= 1 - \left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\sigma}}; \\ \theta^* &= \left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\sigma}}(1 - 2^{-\frac{\lambda}{\sigma}}); \\ \theta^{**} &= 2^{-\frac{\lambda}{\sigma}}\left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\sigma}},\end{aligned}\tag{19}$$

where \underline{n} , n^* and n^{**} are determined as in (13), (8), and (9). Given the Pareto distribution of talent, the average pay of rent-sharing managers is $\underline{w} + \frac{2\lambda}{\lambda-2\sigma}w$, increasing in w by a constant $\frac{2\lambda}{\lambda-2\sigma} > 2$. Then, the following results are obtained.

Proposition 4 *Under Assumption 1, a TFP-enhancing technological improvement will lead to the following consequences:*

- 1) (**Between-group inequality**) *The wage gap between production workers and salaried managers widens.*
- 2) (**Within-group inequality**) *Wage inequality among salaried managers increases.*
- 3) (**Composition of managers**) *If the level of limited-liability restriction is not too high, i.e., $\underline{w} \leq 0$, the share of small business owners among all managers decreases, whereas the shares of both types of salaried managers increase.*

Proof. See Appendix A7. ■

The first two parts of Proposition 4 show that technological progress has a disproportionate impact on the pay level across the three groups of individuals: production workers, lower-level salaried managers, and higher-level salaried managers. Technological progress increases the market wages of production workers. The lower-level managers, who receive a bonus contract and earn an expected wage $2w - \underline{w}$, receive higher pay because of the increase in production workers' wages. The higher-level managers, who are compensated under a rent-sharing contract, receive higher pay because TFP-enhancing technological progress induces the reallocation of production workers from smaller to larger firms.

Note that TFP-enhancing technological progress has an ambiguous effect on the earnings of small business owners. On the one hand, technological progress enables a manager to generate larger surplus. On the other hand, competition from larger firms tends to reduce the size of small businesses. These two conflicting forces complicate the effect of technological progress on the earnings of small business owners.

In the third part of Proposition 4, an additional restriction on \underline{w} is imposed to ensure that a slight change in a manager's outside option (i.e., the wage of production workers) does not trigger an enormous response in the transformation between residual claimants and salaried managers.¹⁶ This can be seen in an extreme case when $\underline{w} = w$ and the entire small

¹⁶The result remains unchanged for a positive level of \underline{w} as long as \underline{w} is not too close to managers' outside

business owner segment disappears. Under this condition, the effect on the transformation between workers and managers dominates the effect on the adjustments between different types of managers. As a result, the share of small business owners decreases, and the share of salaried managers increases. Because of the Pareto distribution, the fraction of rent-sharing managers among salaried managers is independent of the economy-wide technological factor. Therefore, the shares of both types of salaried managers increase.

A rigorous test of the theoretical predictions in this section is beyond the scope of this paper. Nevertheless, I examine Proposition 4 with broad data patterns regarding the wage dynamics and long-term employment in the U.S.. Panel A of Figure 5 plots the managerial wage premium – defined as the difference in the annual real (CPI-adjusted) wages between managers and production/clerical workers – over the 1997-2014 period. The wage premium for salaried managers rises from approximately 20,000 dollars in the 1990s to more than 30,000 dollars in the 2010s. The premium for top managers (CEOs and general managers) more than doubles during the sample period, whereas the premium for mid-level managers (division managers and plant managers) rises far more modestly. Isolating low-level managers (first-line supervisors and administrative managers) shows that their wage premium barely changes over time. These results, particularly the differentiated trends across different types of managers, are consistent with the first two results in Proposition 4, assuming a trend of TFP-enhancing technological progress.

Several pieces of evidence are consistent with the last result in Proposition 4. Panel B of Figure 5 shows that the proportion of the self-employed (a proxy for small business owners) in total employment gradually decreases over the 2003-2011 period, and that the proportion of salaried managers exhibits a similar downward trend.¹⁷ Moreover, the share of mid-level managers among all managers (self-employed and salaried managers combined) increases significantly over time, although the increasing trend in the share of higher-level managers is less obvious.

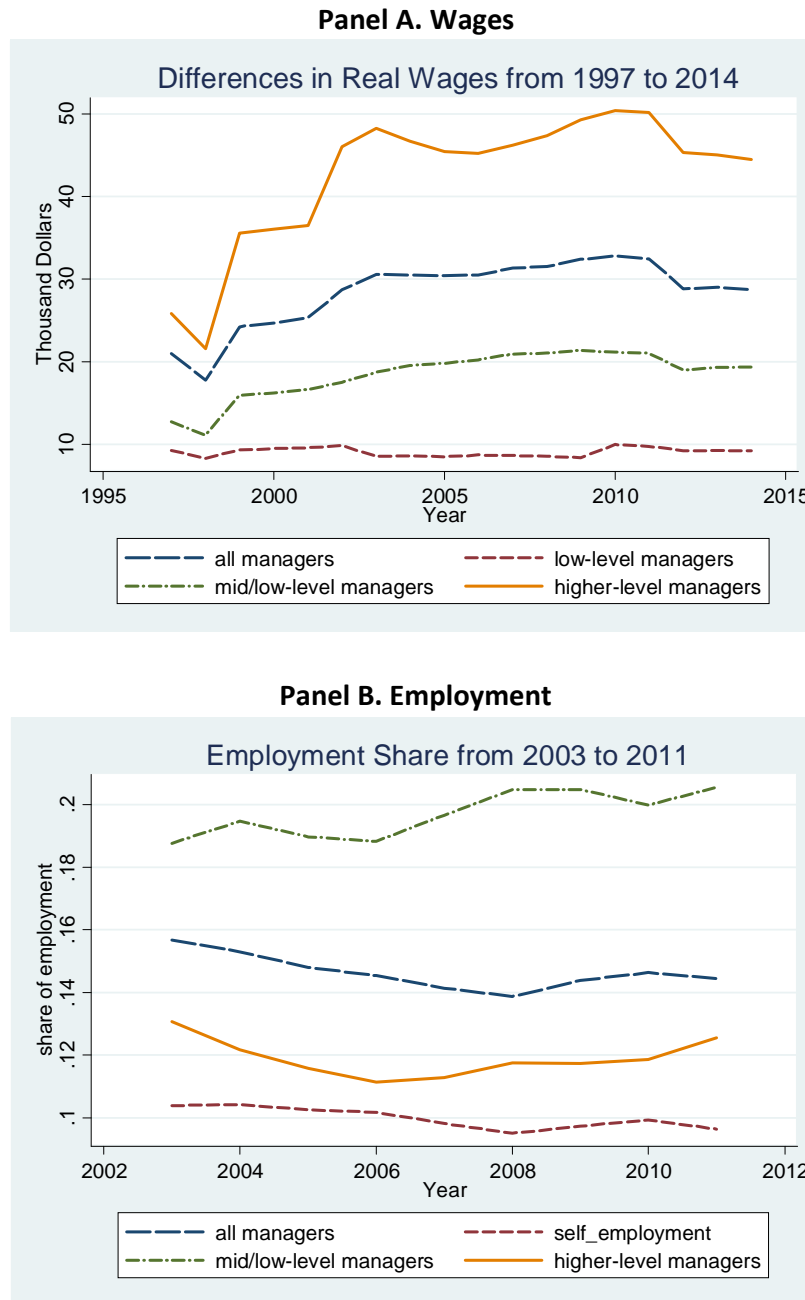
4.2 The Impact of Product Market Competition

The impact of product market competition on managerial incentives is an important topic in the study of innovation and firm performance (e.g., Nickell 1996; Vives 2008), executive behaviour (e.g., Hermalin 1992; Schmidt 1997; Scharfstein 1988; Raith 2003), and managerial compensation (Subramanian 2013; Gersbach and Schmutzler 2014). Prior research mainly focuses on the effects of market competition on a particular type of incentive provision for managers. This paper expands the scope of investigation by simultaneously studying the effects of competition on the selection of managers, the composition of different incentive

option.

¹⁷The decline in the number of self-employed individuals in the U.S. labour market is even more pronounced when the time series is extended back to the 1970s. Data on the employment of various types of managers based on occupation can be obtained from the U.S. National Occupational Compensation Statistics. However, consistent measures are only available for the 2003-2011 period.

Figure 5. Dynamics of Managerial Premium and Employment



Notes: The difference in real wage is defined as the difference between the annual average wage of a particular type of managers and that of production and clerical workers, normalized by the yearly national CPI in the U.S. Low-level managers are individuals whose occupations are described as “first-line supervisors” or “administrative managers.” Top managers are individuals whose occupations are described as “general managers” or “chief executives.” Mid-level managers are individuals whose occupations belong to the managerial category, but are not described as those of the low-level managers or top managers. Data source: US. National Occupational Compensation Statistics, US. Bureau of Labor Statistics.

contracts, and the distribution of wages. To this end, I modify the perfect competition framework in the baseline model to a monopolistic competition framework, while keeping the other features of the baseline model unchanged.

Demand. The economy is abundant in projects, each leading to production of a variety of good, indexed with i . Once produced, the variety i faces a market demand

$$q_i = Qp_i^{-\varepsilon}, \quad (20)$$

where p_i is the price of good i and Q is an endogenous aggregate index that a firm takes as given. The parameter $\varepsilon > 1$ measures the demand elasticity in a market. A larger ε means that consumers are more responsive to price changes. I assume ε to be constant across varieties of goods within a market. Such a demand system with constant demand elasticity can be derived from the Dixit-Stiglitz (1977) preferences. For expositional simplicity, I will drop the variety index i .

Production. The production of a variety features increasing returns to scale. After paying an irreversible fixed operation cost f , a project with productivity a produces q units of product at a marginal cost $\frac{1}{a}$. As in the baseline model, all costs are borne by raw labour provided by production workers. The production of a variety is to maximise the following objective function:

$$\max_p \left(p - \frac{1}{a}\right)q - f. \quad (21)$$

Here, I normalize the wages of production workers to one, while endogenising the aggregate price index.

The above demand and production functions (20) and (21) imply monopolistic competition. The pricing rule for each variety is a constant mark-up over the marginal cost: $p(a) = \frac{\varepsilon}{\varepsilon-1} \frac{1}{a}$. Note that in this setup, a firm corresponds to a variety of goods, and its boundary is determined by the market demand for this good. Such a characterization of firms is standard in the classic monopolistic competition models (e.g., Krugman 1979).

Market Structures. The employment of production workers, revenue, and profit of a good produced with productivity a are:

$$n(a) = Q \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} a^{\varepsilon-1}, \quad (22)$$

$$r(a) = Q \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon+1} a^{\varepsilon-1}, \quad (23)$$

$$\pi(a) = \frac{r(a)}{\varepsilon} - f. \quad (24)$$

The relative employment of production workers and revenues of any two firms with different productivity can be explicitly expressed as:

$$\frac{n(a_i)}{n(a_j)} = \frac{r(a_i)}{r(a_j)} = \left(\frac{a_i}{a_j}\right)^{\varepsilon-1} \text{ for all } i, j. \quad (25)$$

These relationships, analogous to (6) in Section 2, illustrate that a more-productive firm has a larger size in terms of both employment and revenues.

The managerial labour market and the contracting between firm owners and managers remain the same as in Section 2, and the equilibrium analysis in Section 3 directly applies, with several notational modifications. In particular, a triple threshold managerial talent $\{\underline{a}, a^*, a^{**}\}$ stratifies the population into production workers, residual claimants, salaried managers paid with a bonus contract, and salaried managers paid with a rent-sharing contract. These threshold values of talent are defined by the corresponding threshold values of firm size $\{\underline{n}, n^*, n^{**}\}$ such that

$$\frac{a^*}{\underline{a}} = \left[\frac{n^*}{\underline{n}}\right]^{\frac{1}{\varepsilon-1}} \quad \text{and} \quad \frac{a^{**}}{a^*} = 2^{\frac{1}{\varepsilon-1}}, \quad (26)$$

where $\underline{n} = \frac{(\varepsilon-1)[-1+\sqrt{1+2\xi(\varphi^{\varepsilon-1}-1)^2(f+1)}}{\xi(\varphi^{\varepsilon-1}-1)^2}$ and $n^* = \frac{\varepsilon-1}{\varphi^{\varepsilon-1}-1} \sqrt{\frac{2(1-w)}{\xi}}$. The cutoff talent \underline{a} , which divides the population between production workers and managers, is uniquely determined by a labour-market clearing condition similar to (12).

With the equilibrium being determined, the expected pay function $W(a)$ can be written in terms of managerial talent:

$$\begin{aligned} W^{\text{worker}}(a) &= 1 && \text{for } a \in (0, \underline{a}) ; \\ W^{\text{residual}}(a) &= 1 + \frac{\xi}{2} Q^2 (\varphi^{\varepsilon-1} - 1)^2 \varepsilon^{-2\varepsilon} (\varepsilon - 1)^{2(\varepsilon-1)} a^{2(\varepsilon-1)} && \text{for } a \in [\underline{a}, a^*]; \\ W^{\text{bonus}}(a) &= 2 - \underline{w} && \text{for } a \in [a^*, a^{**}); \\ W^{\text{rent}}(a) &= \underline{w} + \frac{\xi}{4} Q^2 (\varphi^{\varepsilon-1} - 1)^2 \varepsilon^{-2\varepsilon} (\varepsilon - 1)^{2(\varepsilon-1)} a^{2(\varepsilon-1)} && \text{for } a \in [a^{**}, \infty). \end{aligned}$$

The form of the above wage function is analogous to that of (15) except that I normalize the wages of production workers to one. The returns to talent for residual claimants and rent-sharing managers are scaled up exponentially by the demand elasticity ε . A sufficiently large ε will yield a convex relationship between managerial pay and managerial talent.

Using the Pareto distribution of talent (Assumption 1), the shares of each type of managers can be written as:

$$\begin{aligned} \underline{\theta} &= 1 - \left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\varepsilon-1}}; \\ \theta^* &= \left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\varepsilon-1}} (1 - 2^{\frac{-\lambda}{\varepsilon-1}}); \\ \theta^{**} &= 2^{\frac{-\lambda}{\varepsilon-1}} \left(\frac{n^*}{\underline{n}}\right)^{-\frac{\lambda}{\varepsilon-1}}. \end{aligned}$$

I consider the impact of an increase in demand elasticity, ε . In the monopolistic competition model, demand elasticity is negatively related to the price-cost margin of a firm; thus, a larger ε implies more intense market competition, i.e., more aggressive interactions

between firms, and firms offering lower prices will attract more consumers away from those offering higher prices. This price competition generates two effects. First, greater demand elasticity enables more-productive firms to steal business from less-productive firms more easily. In consequence, production workers are reallocated from less-talented to more-talented managers. Second, because the gain from a price reduction increases with demand elasticity, firms are willing to offer greater incentives to induce productivity-enhancing managerial effort. These within-firm adjustments lead to further price reductions and amplify the reallocation of production workers across firms. The overall effect yields the following.

Proposition 5 *Under Assumption 1, greater demand elasticity in the product market leads to the following results:*

- 1) (**Selection Effect**) *The talent threshold for becoming a manager increases.*
- 2) (**Wage Inequality**) *The average wage of lower-level salaried managers does not change, whereas the average pay of higher-level salaried managers increases; moreover, wage inequality among higher-level managers increases.*
- 3) (**Composition of Managers**) *The share of small business owners among all managers decreases, whereas the share of salaried managers who share rent with firms increases.*

Proof. See Online Appendix C2.¹⁸ ■

The first part of the above proposition is the positive selection effect resulting from the reallocation of workers from less-talented to more-talented managers. Although the consequence is qualitatively similar to that resulting from technological progress, this selection effect is driven by a rather different mechanism. TFP-enhancing technological progress drives up the wages of production workers, causing those firms that cannot afford the hiring of workers to exit. Here, greater demand elasticity results in a lower output price and more aggressive interactions among firms, culminating in the exit of the firms whose revenues are not sufficient to cover fixed costs.

The second result demonstrates how different incentive contracts transform the effects of product competition. For lower-level managers receiving a bonus contract, the optimal contract will adjust to match their outside option – the market wage for production workers. Unlike technological progress which affects workers’ marginal product and thus their wages, a change in demand elasticity does not affect workers’ wages. In consequence, the pay of lower-level managers is immune to product competition. By contrast, greater demand elasticity increases the pay of higher-level managers who share firm profits. As a result of a more-skewed distribution of rent, wage inequality among higher-level managers also arises. Specifically, the wage distribution within the group of rent-sharing managers is governed by a Pareto distribution with a shape parameter $\frac{\lambda}{2(\varepsilon-1)}$, scaled down from the parameter of the initial

¹⁸The results in this proposition rely on the existence of a unique equilibrium of the model. Therefore, I relegate the proof of this proposition together with the proof of existence to the on-line appendix (Appendix C).

talent distribution by a factor of $\frac{1}{2(\varepsilon-1)}$. Thus, a slight increase in ε may substantially redistribute rent among rent-sharing managers and increase wage inequality among them.

The last result concerns the effect of product competition on the composition of incentive contracts among managers. Because managers' outside option is unaffected by product competition, the restriction $\underline{w} \leq 0$ imposed in the last result of Proposition 4 is no longer necessary. The shrinking group of small business owners and the expanding group of rent-sharing managers precisely reflect the asymmetric effect of product competition on firms with different productivity. Greater demand elasticity facilitates the reallocation of workers across firms, worsening the survival conditions of low-productivity firms, i.e., small businesses. Conversely, it allows the high-productivity firms to steal more businesses from other firms and increases the value of managerial effort, encouraging a larger fraction of firms to use the rent-sharing contract for their managers. Note that the effect on the fraction of salaried managers paid by a bonus contract is ambiguous because their firms steal business from less-productive firms but, in turn, have their business stolen by more-productive firms.

It is notoriously difficult to identify the effect of market competition not only because market competition is often endogenous but also because measuring market competition encompasses various dimensions, including product substitutability, price-cost margins, the level of concentration, and entry costs. In the current monopolistic competition framework, the notion of market competition is captured by the price elasticity of demand, which can be measured by a mark-up over cost. This notion of competition is in line with the traditional measure of market power and with recent empirical research using relative profit differences between firms to study competition in a market with heterogeneous firms (e.g., Boone 2008). Factors that affect demand elasticity include consumer preferences, market thickness, information search costs, transportation costs, and regulation. In this section, I provide some inter-industry evidence to examine the predictions derived from Proposition 5, leaving a systematic investigation of the effect of market competition to future research.

Based on the monopolistic competition framework, several studies (e.g., Hall 1988; Shapiro 1987; Domowitz et al. 1988) have shown substantial variations in market power – measured by the price-cost margin – across broadly-defined industries. Notwithstanding the controversy regarding the methods of estimating market power in a broad industry, a sensible and robust result is that the services industry is considerably more competitive than the manufacturing industry, which in turn is significantly more competitive than the transportation and the public utilities industries. Given the wide application of information technologies in wholesale and retail trade and the emergence of a global supply chain over the last two decades, it seems reasonable to regard the trade industry as highly competitive. Figures B1 and B2 presented in the online appendix compare the wage dynamics and the employment of different types of managers across the following four industries: services, manufacturing, trade, and transportation and utilities combined.

As shown in the first panel of Figure B1, wage inequality between salaried managers and

workers in the transportation and utilities industries is considerably less pronounced than in the other three industries. The next two panels demonstrate that these inter-industry differences in the between-group inequalities is driven primarily by wage inequality between top managers and workers: during the 2010s, the ratio of the annual wage of top managers to that of workers is greater than 4 in the manufacturing and services industries but below 3.5 in the transportation and utilities industries. Wage inequality between mid- and low-level managers and workers is stable over time and across industries, which is consistent with the theoretical prediction that the pay of lower-level managers does not vary with individual ability and firm profitability. The last panel plots wage inequality within managers, as measured by the wage ratio between top managers and low-level managers. Consistent with Proposition 5, this within-group inequality in the services industry is greater than in the manufacturing and trade industries, which is in turn greater than in the transportation and utilities industries.

With respect to employment, the model predicts that in industries characterized by more intensive competition or a lower price-cost margin, the share of managers as a whole and the share of the self-employed in the entire working population should be smaller, whereas the share of top managers either among all managers or among salaried managers should be larger. Consistent with these predictions, Figure B2 shows that both the share of managers and the share of the self-employed among all industry workers in the manufacturing industry are substantial smaller than in the transportation and utilities industries, whereas the share of top managers among all managers or among salaried managers is substantially larger. A similar pattern, although less pronounced, is also revealed in the trade industry. However, the employment pattern in the services industry seems to conflict with the theoretical prediction, which may result from the exceptionally large number of self-employed workers in that industry.

The above inter-industry evidence is merely suggestive. Using micro-level data, several studies (e.g., Hubbard and Palia 1995; Cuñat and Guadalupe 2005, 2009; Karuna 2007) have shown that market competition tends to induce firms to provide more managerial incentives. Along this line of research, further empirical investigation should examine both the heterogeneous effect and the composition effect of market competition.

5 Discussion and Extensions

In the baseline model, several assumptions restrict a manager's outside option. Specifically, a manager, after employed, is fixed within an employment relationship; managers cannot establish their own firms; and individuals are not endowed with any wealth. These assumptions substantially simplify the contracting process and provide a convenient way to embed agency problems within firms in a general equilibrium model. In this section, I examine how the results change when these assumptions are relaxed.

5.1 Labour Market Frictions

In the model, the labour market for managers is assumed to be perfectly immobile. Thus, the owners have all the bargaining power. In contrast, under the competitive assignment approach, the opposite assumption holds: managers are perfectly mobile across firms, which offer a wage equal to the surplus created by the next most talented manager. I offer a simple compromise between the two extreme assumptions by allowing managers to conduct costly search in the labour market.

In addition to the basic setup in Section 2, suppose, with probability δ , an individual who is searching for a managerial job can successfully reveal her talent to the market before contracting with an anonymous capitalist. For instance, before entering the managerial labour market, a manager can obtain a certificate that will help convince potential employers of her ability to improve firm productivity. Given that capitalists are abundant and ex ante identical, such a "certified" manager has all the bargaining power and can obtain the entire surplus created in the employment relationship. Therefore, at the time of contracting, a manager's outside option is $\max\{w, \delta V^{FE}(a)\}$, where $V^{FE}(a)$ is the expected surplus created by a manager who exerts the first-best effort that stochastically improves firm productivity.¹⁹

In this case, a richer sorting pattern emerges.

Corollary 1 *Suppose that the probability of revealing one's talent to the market is positive but sufficiently small. In equilibrium, there exists a quadruple of threshold managerial talent levels $\{\underline{a}, a^*, a^{**}, \bar{a}\}$ in increasing order such that: 1) individuals with managerial talent below \underline{a} are production workers; 2) individuals with talent between \underline{a} and a^* are residual claimants; 3) individuals with talent between a^* and a^{**} are salaried managers whose pay is invariant with talent; 4) individuals with talent between a^{**} and \bar{a} are salaried managers who share rent with employers; 5) individuals with talent above \bar{a} are entrepreneurial managers who claim residuals of the firms.*

Proof. This corollary is a minor extension of Lemma 4 and Proposition 2, and the proof is thus omitted. ■

Because of the opportunity to reap the entire surplus accruing to their own talent and effort, the most-talented individuals will continue to search in the managerial labour market until their talent is revealed and capitalists compete for them. Therefore, a new type of residual claimant – the superstar entrepreneurial manager – exists in equilibrium, in addition to the low-talented individuals who passively become owners of small businesses. The presence of entrepreneurial managers has two additional effects on the wage distribution.

¹⁹Here, I assume that a manager who decides but fails to reveal his ability to the owner will obtain a zero payoff. This is the cost of giving up being a production worker. Another interpretation is that in order to reveal ability, a manager ought to pay a certification or searching cost that is proportional to the future value of his or her talent. Alternatively, a manager's outside option can be written as $\max\{w, \delta V^{FE}(a) + (1 - \delta)w\}$, which implies that a manager who fails to reveal ability can enter the market for production workers without any costs. The qualitative results presented in this subsection will remain unchanged.

First, it strengthens the superstar effect, as entrepreneurial managers obtain greater surplus than salaried managers. Second, with less-efficient incentive structures being replaced by more-efficient ones, competition between firms for raw labour is intensified, and it is more difficult to enter the managerial occupation. As a result, both the wage inequality between managers and workers and the wage dispersion among managers increase, compared to the case without superstar managerial entrepreneurs.

The sorting pattern in Corollary 1 depends crucially on the parameter δ . When $\delta = 0$, it is prohibitively costly to reveal one's ability to the market, and a manager's outside option is merely production workers' wages. The model becomes the same as the baseline model in which no entrepreneurial managers exist in equilibrium. When δ rises, more and more individuals will become entrepreneurial managers. In the extreme case when $\delta = 1$, the model can be seen through the lens of competitive assignment: abundant homogeneous capitalists compete for a limited number of managers with heterogeneous talent. All managers then become residual claimants. In this sense, a perfect managerial labour market can curb agency problems within firms.

The parameter δ can be also interpreted as a measure of general human capital. A larger δ implies that managerial talent is less specific to firms. More general managerial talent (human capital) and thus a more mobile managerial labour market allow for more efficient incentive structures, which enhances the returns to top talent and increases wage inequality. This is consistent with the arguments that attribute the surge in CEO pay to an increase in the importance of general managerial ability, relative to firm-specific human capital (e.g., Murphy and Zabojnik 2004, 2007; Frydman 2007).

5.2 Barriers to Entrepreneurship

In the baseline model, the limited-liability constraint entails that more-talented individuals become salaried managers rather than business owners. The result hinges on the assumption that a manager cannot establish their own businesses. One justification of this assumption is the existence of barriers to entrepreneurship – the difficulty of financing start-ups, regulatory obstacles such as entry fees and bureaucratic procedures, and the likelihood of being expropriated. I now introduce the possibility of overcoming these barriers to entrepreneurship into the baseline model.

Specifically, I assume that after paying a fixed cost f_e , an individual can successfully develop a new project and manage the project by herself. Being a managerial entrepreneur, the individual exerts the first-best level effort e^{FB} and claims the entire surplus of the firm. The participation constraint in the optimization problem (3) becomes:

$$eb(a) - \frac{1}{2\xi}e^2 + s(a) \geq \max\{w, V^{FB}(a) - f_e\},$$

where $V^{FB}(a)$ denotes the expected firm profit when a manager with talent a exerts the first-best effort. The baseline model is a special case in which f_e is extremely large. Conversely, when $f_e \rightarrow 0$, all managers develop their own projects; the allocation of talent is a simple dichotomy between workers and entrepreneurs. When the entrepreneurial entry cost f_e lies in between these two extreme cases, but is sufficiently large so that only the most-talented individuals become entrepreneurial managers, a sorting pattern similar to the result in Corollary 1 emerges.

5.3 Limited Liability Constraint

The above two extensions focus on managers' outside options. Another extension is to allow for variation in the limited liability constraint, \underline{w} , which is a key driver of the model. In the baseline model, I assume that \underline{w} is smaller than managers' outside option w but is bounded from below. This assumption generates the distinction between small business owners and salaried managers. On the one hand, a larger \underline{w} will crowd out small business owners and expand the group of salaried managers, and more-efficient contracts will be replaced by less-efficient contracts. On the other hand, if $\underline{w} \rightarrow -\infty$, an owner can always "sell the store" to a manager; the population is sorted into production workers and business owners. As a result, the economy achieves the first-best efficiency at the cost of a higher level of wage inequality.

One way to introduce heterogeneity in the limited liability constraint is to allow individuals to differ in their wealth. The introduction of wealth would complicate the model in two ways. First, the wealth allows project owners to "sell the store" more easily, and thus more individuals will become small business owners. Second, endowed with wealth, an individual is more capable of starting up her own enterprise, resulting in more entrepreneurial managers in the economy. These two forces, which work on different margins, move in the same direction to crowd out salaried managers and improve the efficiency of the economy. In the current model without wealth endowment, these results could be obtained in a dynamic setting in which individuals can accumulate wealth. I leave elaboration along this line to future research.

6 Conclusion

The allocation of heterogeneous individuals to different reward structures is an important and complex economic issue. Preferences, cognitive ability, job characteristics, and firm productivity may all affect the matching of individuals and reward structures. This paper singles out one dimension of heterogeneity – the talent for managing productive resources – and analyses its interaction with one type of contractual friction – limited liability. Starting with an exogenous distribution of managerial talent, the model endogenises the distribution of firm size, the distribution of optimal incentive contracts, the division of surplus within firms, and ultimately the distribution of wage earnings.

Two elements are key to the analysis: 1) a managerial firm in which the provision of managerial incentives is costly due to limited liability and 2) an endogenous firm-size distribution that determines the value of managerial talent. The interaction between these two elements generates a two-stage selection of incentive structures. First, driven by the limited-liability constraint, a division emerges between residual claimants and salaried managers. Second, driven by the participation constraint, two types of salaried managers can be distinguished: one paid a bonus contract that is invariant with individual ability and firm profitability and the other paid a rent-sharing contract. The equilibrium features an interesting sorting pattern: within two groups – the least-talented and medium-talent individuals, the incentive structure equalizes the pay level within each group; among the other two groups – the low-talent and high-talent individuals, the incentive structure amplifies the returns to talent. This result addresses some empirically observed relationships between incentive structures and pay levels both within and across managerial groups.

The framework developed in this paper is highly tractable and can be used to analyze how technological progress, demand shocks, and changes in market structure affect managerial efficiency and returns to skills. In a separate paper (Wu 2013), I extend the current model to an open economy and derive some novel implications regarding the relationship between trade liberalisation and wage inequality.

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7 Appendix A

A1. Proof of Lemma 1.

Proof. In the constrained-optimization problem (3), $s(a)$ is a lump-sum transfer that does not distort incentives. Suppose that PC is slack. If $s(a) > \underline{w}$, the principal (i.e., the owner) can increase payoffs by reducing $s(a)$ by a small amount. Suppose that WC is slack. If PC is slack, the principal can be better off by reducing $s(a)$ slightly so that the manager is still willing to participate. Hence, at least one of the two constraints should bind at optimum. ■

A2. Proof of Lemma 2.

Proof. Suppose that WC is slack. By Lemma 1, PC must bind. The principal "sells the store" to the manager at the price $t(a) = \{e^{FB}(a)\pi(\varphi a) + [1 - e^{FB}(a)]\pi(a) - \frac{(e^{FB}(a))^2}{2\xi}\} - w$, where $e^{FB}(a)$ is defined in (7). The condition for attaining this is that WC is not binding, even in the low state: $s(a) = \pi(a) - t(a) > \underline{w}$ or $\frac{\xi}{2}[\pi(\varphi a) - \pi(a)]^2 = \frac{\xi}{2}\Phi^2\pi(a)^2 < w - \underline{w}$. ■

A3. Proof of Lemma 3.

Proof. Because the incentive compatibility constraint (IC) is a concave function in e , one can replace it with the first-order condition $e = \xi b(a)$. Substitute this into the principal's objective function to obtain the value function: $V(a) = e[\pi(\varphi a) - b(a)] + (1 - e)\pi(a) - s(a)$. Given that WC is binding, $s(a) = \underline{w}$.

Case 1: If PC is binding, I denote all solutions with the superscript BP . The net payoff accruing to the manager is $\xi b(a)^2 - \frac{1}{2\xi}\xi^2 b(a)^2 + \underline{w} = w$, from which I solve for $b^{BP}(a) = \sqrt{\frac{2(w-\underline{w})}{\xi}}$ and $e^{BP} = \sqrt{2\xi(w-\underline{w})}$. An interior solution of $e^{BP} \in (0, 1)$ is guaranteed under the restriction $1 - \frac{1}{2\xi} < \underline{w} < w$.

Case 2: If PC is slack, I denote all solutions with the superscript RP . Substitute $e = \xi b(a)$ and $s(a) = \underline{w}$ into the objective function. Solving the optimization problem, $e^{RP}(a) = \frac{\xi[\pi(\varphi a) - \pi(a)]}{2} = \frac{\xi\Phi}{2}\pi(a)$ and $b^{RP}(a) = \frac{\Phi}{2}\pi(a)$. ■

A4. Proof of Proposition 1.

Proof. Case 1: $n(a) < n^*$. The condition in Lemma 2 is satisfied and the first-best effort is achievable. Obviously, $V^{sell}[n(a)] > V^{rent}[n(a)]$. And $V^{sell}[n(a)] - V^{bonus}[n(a)] = [\sqrt{\frac{\xi}{2}}\Phi\frac{wn(a)}{\sigma-1} - (w - \underline{w})]^2 \geq 0$ with equality at $n(a) = n^*$. Hence, the firm will choose the sell-the-store contract.

Case 2: $n^* \leq n(a) < n^{**}$. The condition in Lemma 2 is violated and the first-best effort is not feasible. If the manager accepts the rent-sharing contract, her net payoff would be $\frac{\xi}{\sigma}(\frac{\Phi}{\sigma-1})^2 w^2 n(a)^2 + \underline{w}$, which is smaller than her outside option when $n(a) < n^{**}$. Therefore, both the wealth constraint and the participation constraint are binding. The owner will choose the one-step-bonus contract, in which case $e^{BP} = \sqrt{2\xi(w - \underline{w})} \geq e^{RP}(a) = \frac{\xi}{2}\Phi\pi(a)$.

Case 3: $n(a) \geq n^{**}$. Now, the wealth constraint is binding while the participation constraint is not. $V^{rent}[n(a)] - V^{bonus}[n(a)] = [\frac{\sqrt{\xi}}{2}\Phi\frac{wn(a)}{\sigma-1} - \sqrt{2(w - \underline{w})}]^2 \geq 0$ with equality at $n(a) = n^{**}$. Hence, the owner will choose the rent-sharing contract. ■

A5. Proof of Lemma 4.

Proof. The three equilibrium equations are reproduced as follows:

$$\underline{n} \equiv n(\underline{a}) = \frac{\sigma - 1}{w} \frac{-1 + \sqrt{1 + 2w\xi\Phi^2}}{\xi\Phi^2}; \quad (\text{A1})$$

$$w = \frac{\sigma - 1}{\sigma} \underline{a} \underline{n}^{-\frac{1}{\sigma}}; \quad (\text{A2})$$

$$\int_{\underline{a}}^{\infty} \{[e(a)n(\varphi a) + [1 - e(a)]n(a)]g(a)da = \int_0^{\underline{a}} g(a)da. \quad (\text{A3})$$

With these equations, I will prove this lemma in two steps. First, I take \underline{a} as given and show that \underline{n} and w are uniquely determined. Second, I show that \underline{a} is uniquely pinned down by a given pair of $\{\underline{n}, w\}$.

Step 1. For a given \underline{a} , a pair of (\underline{n}, w) can be calculated from (A1) and (A2).

Step 2. The right-hand side of (A3) is the labour supply, which obviously increases in \underline{a} . I now show that the left-hand side of (A3) – the labour demand – is decreasing in \underline{a} for any given w . Define

$$\begin{aligned} L^D(\underline{a}) &= \int_{\underline{a}}^{\infty} \{[e(a)n(\varphi a) + [1 - e(a)]n(a)]g(a)da \\ &= \int_{\underline{a}}^{\infty} n(a)g(a)da + \Phi \int_{\underline{a}}^{a^*} e(a)n(a)g(a)da + \Phi \int_{a^*}^{a^{**}} e^*n(a)g(a)da + \Phi \int_{a^{**}}^{\infty} e(a)n(a)g(a)da. \end{aligned}$$

Substituting into the above expression the optimal managerial efforts and $n(a) = (\frac{a}{\underline{a}})^{\sigma} \underline{n}$

obtained from (6) and using (14), the following is obtained:

$$L^D(\underline{a}) = \underline{n} \int_{\underline{a}}^{\infty} \left(\frac{a}{\underline{a}}\right)^{\sigma} g(a) da + \Phi \left\{ \frac{\xi \Phi}{\sigma - 1} w \underline{n}^2 \int_{\underline{a}}^{a^*} \left(\frac{a}{\underline{a}}\right)^{2\sigma} g(a) da \right. \\ \left. + \sqrt{2\xi(w - \underline{w})} \underline{n} \int_{a^*}^{a^{**}} \left(\frac{a}{\underline{a}}\right)^{\sigma} g(a) da + \frac{\xi \Phi}{2(\sigma - 1)} w \underline{n}^2 \int_{a^{**}}^{\infty} \left(\frac{a}{\underline{a}}\right)^{2\sigma} g(a) da \right\}.$$

Then,

$$\frac{dL^D}{d\underline{a}} = - \int_{\underline{a}}^{\infty} \frac{\sigma}{\underline{a}} \left(\frac{a}{\underline{a}}\right)^{\frac{1}{1-\beta}} \underline{n} g(a) da - g(\underline{a}) + \Phi \left\{ \frac{\xi \Phi}{\sigma - 1} w \underline{n}^2 \left(\frac{a^*}{\underline{a}}\right)^{2\sigma} g(a^*) \frac{da^*}{d\underline{a}} \right. \\ - \frac{\xi \Phi}{\sigma - 1} w \underline{n}^2 g(\underline{a}) - \frac{\xi \Phi}{\sigma - 1} w \underline{n}^2 \int_{\underline{a}}^{a^*} \frac{2\sigma}{\underline{a}} \left(\frac{a}{\underline{a}}\right)^{\frac{2}{1-\beta}} g(a) da \\ + \sqrt{2\xi(w - \underline{w})} \underline{n} \left(\frac{a^{**}}{\underline{a}}\right)^{\sigma} g(a^{**}) \frac{da^{**}}{d\underline{a}} - \sqrt{2\xi(w - \underline{w})} \underline{n} \left(\frac{a^*}{\underline{a}}\right)^{\sigma} g(a^*) \frac{da^*}{d\underline{a}} \\ \left. - \frac{\xi \Phi}{2(\sigma - 1)} w \underline{n}^2 \left(\frac{a^{**}}{\underline{a}}\right)^{2\sigma} g(a^{**}) \frac{da^{**}}{d\underline{a}} - \frac{\xi \Phi}{2(\sigma - 1)} w \underline{n}^2 \int_{a^{**}}^{\infty} \frac{2\sigma}{\underline{a}} \left(\frac{a}{\underline{a}}\right)^{\frac{2}{1-\beta}} g(a) da \right\}.$$

The terms involving the derivatives of the threshold values of talent cancel out after substituting (14):

$$\left(\frac{a^*}{\underline{a}}\right)^{\sigma} \underline{n} g(a^*) \left[\frac{\xi \Phi}{\sigma - 1} w \underline{n} \left(\frac{a^*}{\underline{a}}\right)^{\sigma} - \sqrt{2\xi(w - \underline{w})} \right] \frac{da^*}{d\underline{a}} \\ + \left(\frac{a^{**}}{\underline{a}}\right)^{\sigma} \underline{n} g(a^{**}) \left[\sqrt{2\xi(w - \underline{w})} - \frac{\xi \Phi}{2(\sigma - 1)} w \underline{n} \left(\frac{a^{**}}{\underline{a}}\right)^{\sigma} \right] \frac{da^{**}}{d\underline{a}} = 0.$$

Therefore, $\frac{dL^D}{d\underline{a}} < 0$. This downward-sloping demand curve and the upward-sloping labour supply curve guarantee single crossing, and a unique \underline{a} is determined for any given w .

In the above proof, I use the fixed-point argument to show existence of the equilibrium. Uniqueness of the equilibrium can be shown by substituting (A1) into (A2) to obtain an equation in (\underline{a}, w) . Together with equation (A3), it can be shown that a unique pair of equilibrium (\underline{a}, w) is determined by two single-crossing curves in the (\underline{a}, w) space. ■

A6. Proof of Proposition 3.

Proof. I measure the power of incentives using the sensitivity of pay with respect to firm profit, defined as the ratio of the covariance between individual pay and firm profit to the variance of firm profit: $\gamma(a) = \frac{\text{cov}[b(a), \pi(a)]}{\text{var}(\pi)}$.

Case 1): $a < \underline{a}$. By Proposition 2, these individuals are production workers. They receive a flat wage w , which is independent of firm profit: $\gamma(a) = 0$.

Case 2): $a \in [\underline{a}, a^*]$. These individuals are residual claimants; their pay is perfectly correlated

with firm profit: $\gamma(a) = 1$.

Case 3): $a \in [a^*, a^{**})$. These individuals receive a bonus contract. The covariance and variance components are respectively:

$$\begin{aligned} cov^{bonus}[b(a), \pi(a)] &= \sqrt{2\xi(w - \underline{w})} \left[\sqrt{\frac{2(w - \underline{w})}{\xi}} - 2(w - \underline{w}) \right] [\pi(\varphi a) - \sqrt{2\xi(w - \underline{w})} \Phi \pi(a) - \pi(a)] \\ &\quad + [1 - \sqrt{2\xi(w - \underline{w})}] [-2(w - \underline{w})] [\pi(a) - \sqrt{2\xi(w - \underline{w})} \Phi \pi(a) - \pi(a)] \\ &= 2(w - \underline{w}) [1 - \sqrt{2\xi(w - \underline{w})}] \Phi \pi(a); \end{aligned}$$

$$\begin{aligned} var^{bonus}[\pi(a)] &= \sqrt{2\xi(w - \underline{w})} [\pi(\varphi a) - \sqrt{2\xi(w - \underline{w})} \Phi \pi(a) - \pi(a)]^2 \\ &\quad + [1 - \sqrt{2\xi(w - \underline{w})}] [-\sqrt{2\xi(w - \underline{w})} \Phi \pi(a)]^2 \\ &= \sqrt{2\xi(w - \underline{w})} [1 - \sqrt{2\xi(w - \underline{w})}] [\Phi \pi(a)]^2. \end{aligned}$$

Then,

$$\begin{aligned} \gamma^{bonus}(a) &= \frac{cov^{bonus}[b(a), \pi(a)]}{var^{bonus}[\pi(a)]} \\ &= \frac{\sqrt{2(w - \underline{w})/\xi}}{\Phi \pi(a)} \leq 1. \end{aligned}$$

By the definition of a^* and a^{**} , $\gamma^{bonus}(a)$ decreases in $a \in [a^*, a^{**})$ with $\gamma^{bonus}(a^*) = 1$ and $\gamma^{bonus}(a^{**}) = 0.5$.

Case 4): $a \in [a^{**}, \infty)$. These individuals receive a rent-sharing contract. The covariance and variance components are respectively:

$$\begin{aligned} cov^{rent}[b(a), \pi(a)] &= \frac{\xi \Phi}{2} \pi(a) \left[\frac{\Phi}{2} \pi(a) - \frac{\xi \Phi^2}{4} \pi^2(a) \right] [\pi(\varphi a) - \frac{\xi \Phi^2}{2} \pi^2(a) - \pi(a)] \\ &\quad + \left[1 - \frac{\xi \Phi}{2} \pi(a) \right] \left[-\frac{\xi \Phi^2}{4} \pi^2(a) \right] [\pi(a) - \frac{\xi \Phi^2}{2} \pi^2(a) - \pi(a)] \\ &= \frac{\xi \Phi^2}{4} \pi(a)^2 \left[1 - \frac{\xi}{2} \Phi \pi(a) \right] \Phi \pi(a); \end{aligned}$$

$$\begin{aligned} var^{rent}(\pi) &= \frac{\xi \Phi}{2} \pi(a) \left[\frac{\Phi}{2} \pi(a) - \frac{\xi}{2} \Phi^2 \pi^2(a) - \pi(a) \right]^2 \\ &\quad + \left[1 - \frac{\xi \Phi}{2} \pi(a) \right] \left[\frac{\xi \Phi}{2} \Phi \pi(a) \pi(a) \right]^2 \\ &= \frac{\xi \Phi}{2} \pi(a) \left[1 - \frac{\xi \Phi}{2} \pi(a) \right] \Phi^2 \pi^2(a). \end{aligned}$$

Then,

$$\begin{aligned}\gamma^{rent}(a) &= \frac{\frac{\xi\Phi^2}{4}\pi(a)^2[1 - \frac{\xi}{2}\Phi\pi(a)]\Phi\pi(a)}{\frac{\xi\Phi}{2}\pi(a)[1 - \frac{\xi\Phi}{2}\pi(a)]\Phi^2\pi^2(a)} \\ &= \frac{1}{2} \leq \gamma^{bonus}(a).\end{aligned}$$

The proof of the result that the size of incentives weakly increases in talent directly follows the proof of Propositions 1 and Lemma 4. ■

A7. Proof of Proposition 4.

Proof. With the Pareto distribution, the average wage of the rent-sharing managers is

$$\widetilde{W}^{rent} = \underline{w} + \frac{2\lambda}{\lambda - 2\sigma}w.$$

Therefore, the average wage of all salaried managers is

$$\begin{aligned}\widetilde{W}^{SM} &= \frac{\theta^*}{\theta^* + \theta^{**}}(2w - \underline{w}) + \frac{\theta^{**}}{\theta^* + \theta^{**}} \frac{2\lambda}{\lambda - 2\sigma}w \\ &= (1 - 2^{-\frac{\lambda}{\sigma-1}})(2w - \underline{w}) + 2^{-\frac{\lambda}{\sigma-1}} \frac{2\lambda}{\lambda - 2\sigma}w.\end{aligned}$$

Then,

$$\begin{aligned}\frac{d(\widetilde{W}^{SM} - w)}{dw} &= 2 + 2^{-\frac{\lambda}{\sigma-1}} \frac{4\sigma}{\lambda - 2\sigma} > 0.\end{aligned}$$

The wage distribution of the rent-sharing managers is characterized by a constant \underline{w} and a Pareto distribution with a shape parameter $\frac{\lambda}{2\sigma}$ and an minimum value of $2w$. Since the shape parameter fully determines the variance and the Gini coefficient of a Pareto distribution, the wage inequality among the rent-sharing managers is independent of technological factors. Moreover, given that $\frac{\theta^*}{\theta^* + \theta^{**}} = 1 - 2^{-\frac{\lambda}{\sigma-1}}$ is invariant to technology factors, the wage inequality among salaried managers is fully captured by

$$(w + \frac{2\lambda}{\lambda - 2\sigma})w - (2w - \underline{w}),$$

which increases in w with a factor $\frac{4\sigma}{\lambda - 2\sigma}$.

I now prove the last result of Proposition 4. With the Pareto distribution of talent, the effect of technological progress is only through the market wage for production workers. Since

$$\frac{n^*}{n} = \frac{\Phi\sqrt{2\xi(w-\underline{w})}}{-1 + \sqrt{1 + 2w\xi\Phi^2}},$$

$$\frac{d\frac{n^*}{n}}{dw} = \frac{\Phi\sqrt{2\xi}}{\sqrt{1 + 2w\xi\Phi^2}\sqrt{(w-\underline{w})}} \frac{1 + 2\Phi^2\xi w - \sqrt{1 + 2w\xi\Phi^2}}{(-1 + \sqrt{1 + 2w\xi\Phi^2})^2}.$$

When $\underline{w} = 0$, $\frac{1+2\Phi^2\xi\underline{w}-\sqrt{1+2\underline{w}\xi\Phi^2}}{(-1+\sqrt{1+2\underline{w}\xi\Phi^2})^2} = \frac{1}{1-\sqrt{1+2\underline{w}\xi\Phi^2}} < 0$. Since $\frac{1+2\Phi^2\xi\underline{w}-\sqrt{1+2\underline{w}\xi\Phi^2}}{(-1+\sqrt{1+2\underline{w}\xi\Phi^2})^2}$ increases in \underline{w} , $\frac{d\underline{v}^*}{d\underline{w}} < 0$ when $\underline{w} \leq 0$. By Lemma 5 ($\frac{d\underline{w}}{dt} > 0$), $\frac{d\underline{v}^*}{dt} = \frac{d\underline{v}^*}{d\underline{w}} \frac{d\underline{w}}{dt} < 0$. Therefore, $\frac{d\underline{\theta}}{dt} < 0$, $\frac{d\underline{\theta}^*}{dt} > 0$, and $\frac{d\underline{\theta}^{**}}{dt} > 0$. ■